Effects of Link Rate Assignment on the Max-Min Fair Throughput of Wireless Mesh Networks

Barbara Staehle, Dirk Staehle, and Rastin Pries
University of Würzburg, Institute of Computer Science, Würzburg Germany
{bstaehle, dstaehle, pries}@informatik.uni-wuerzburg.de

***Abstract*—Wireless mesh networks (WMNs) support a variety of modulation and coding schemes (MCSs) and hence enable adaptive modulation and coding. On the one hand, adaptive modulation and coding significantly increases the overall system capacity, as it allows to maximize the throughput on each link in dependence of the channel conditions. On the other hand, there is a trade-off between link data rate and spatial reuse, as high data rate links are less robust against interference. For maximizing the wireless mesh network performance, an intelligent link rate assignment strategy is thus required. By evaluating the max-min fair WMN throughput under different link rate assignment strategies, we are able to show that using more robust MCSs improve the overall WMN throughput and point out directions towards an optimized MCS choice.**

***Index Terms*—Wireless Mesh Networks, Adaptive Modulation and Coding, Max-Min Fairness, Planning**

I. Introduction

The key characteristics of wireless mesh networks (WMNs) are dynamic self-organization, self-configuration, and self-healing [1]. This makes them a promising solution for easy and fast, reliable and cost-effective wireless network deployment in all kinds of environments. Thus, WMNs have not only become an active research topic, but are increasingly used as an Internet access network in private neighborhoods, small companies or cities. In those mesh networks, which we will consider in the remainder of this work, the mesh nodes provide Internet access to clients by forwarding traffic from and to a subset of mesh nodes which serve as gateways to the Internet.

As nodes far away from the gateways rely on nodes closer to the gateways for forwarding their traffic, policies for guaranteeing a minimum amount of bandwidth for all nodes must exist in each mesh network. In an earlier study we focused on the problem of determining the WMN throughput under max-min fair flow rate assignment [2]. A flow rate assignment is called max-min fair, if no rate of a flow can be increased without decreasing another one. The minimum throughput of a flow in the network is thus maximized [3]. Note that this theoretical capacity gives an upper bound for the maximal achievable per flow throughput, which can only be reached by a perfect MAC algorithm and scheduling scheme. In [2] we proposed algorithms for calculating the max-min fair throughput of end-to-end flows in WMNs with heterogeneous link rates which we also use in the remainder of this work.

Another way to increase the capacity of mesh networks is to carefully plan and optimize the WMN deployment. In this study we examine a factor which has not yet been used for planning and optimizing wireless mesh networks. State-of-the art wireless communication standards (e.g. IEEE 802.11 or IEEE 802.16) provide several modulation and coding schemes (MCS) which allow transmissions at different data rates at the price of different degrees of sensitivity to interferences. Most commonly, two nodes are assumed to communicate at the highest possible data rate. If, in contrast, an MCS with a smaller data rate is used for link x, this decreases the link rate, but could allow the use of x in parallel to a neighboring link, hence increase the throughput of the end-to-end data flows. This trade-off between link data rate and spatial reuse has been neglected by most previous analytical works, as oversimplified physical layer models are not able to capture the effects of adaptive modulation and coding on the channel level.

In this work we introduce a more realistic channel abstraction and make our results broadly applicable, by analyzing the sensitivity of MCS optimization to routing mechanisms and network density. Our experiments demonstrate the inherent optimization potential of MCS assignment and allow to point out directions to optimized link rate assignment strategies. Our work is structured as follows: In Section II we formally describe the problem we are going to investigate and summarize existing and our new contributions. Section III contains a description of the used methodology. Numerical results are presented in Section IV. Section V concludes our work and gives an outlook on future research directions.

II. Problem Formulation

This paper aims at examining the influence of MCS or link rate assignment strategies on the capacity of mesh networks. As a measure for the WMN capacity, we use the achievable max-min fair share throughput. We define a mesh network as a set of mesh nodes $\mathcal{N}$ and a set of links $\mathcal{L}$ connecting the nodes. A subset $\mathcal{G} \subseteq \mathcal{N}$ contains the gateway nodes that are connected to the Internet. If a direct communication between nodes i and j is possible, the link (i, j) exists and $r_{i,j}$ is its data rate. All nodes except the gateway nodes are assumed to serve as access points for client devices and are furthermore assumed to have a saturated best-effort data flow from the Internet. For the sake of simplicity of notation we consider only downlink data flows. Including uplink data flows is possible but increases the computational complexity. We denote the direction from the Internet to the mesh nodes as *downlink* and the reverse direction as *uplink*. $\mathcal{F} = \mathcal{N} \setminus \mathcal{G}$ denotes the set of non-gateway nodes having a data flow. Each node $i \in \mathcal{F}$ is connected to the Internet via a fixed path to one gateway. Furthermore, we consider a multi-channel, multi-radio mesh network, i.e. every link (i, j) is assigned a channel $q_{i,j}$ out of a set $\mathcal{Q}$ of non-overlapping channels. The goal is now to compare the max-min-fair end-to-end flow throughputs $\tau(i)$ for every $i \in \mathcal{F}$ which result from different link rate assignment strategies.

In Section II-A we review approaches which are related to this problem and results which we use for our work. Our own contribution is presented in Section II-B.

A. Related work

To estimate the capacity of a wireless network it is crucial to know which nodes can successfully communicate and which transmissions can concurrently take place. One of the first authors studying the wireless network capacity was Abramson in 1970 [4]. The network where his newly introduced random channel access protocol ALOHA was running was a star topology with stations transmitting packets to a central coordinator in an unsynchronized manner. If two stations transmitted at the same time, the packets collided and had to be sent again.

Two models more suitable for characterizing the successful reception of a transmission in a wireless multi-hop network were introduced by Gupta and Kumar [5]. Under the *protocol model*, a transmission from node i to node j succeeds, if the distance between i and j is smaller than a transmission range r and if no other node which is closer to i than $(1+\epsilon)r, \epsilon > 0$ is transmitting at the same time. According to the less abstract *physical model*, a transmission from i is successfully received by j if the *signal to interference and noise ratio* (SINR) $\gamma_{i,j}$ is greater or equal than a threshold $\gamma^*_{i,j}$:

$$\gamma_{i,j} = \frac{R_{i,j}}{N+I_{i,j}} \geq \gamma^*_{i,j}. \tag{1}$$

$R_{i,j}$ is the power received at j when i is transmitting and N is the ambient noise power [5]. The interference $I_{i,j}$ is calculated as the power received from nodes $a \in \alpha$ which are transmitting on channel $q_{i,j}$ at the same time, as i: $I_{i,j} = \sum_{a\in\alpha} R_{a,j}$.

For networks with n randomly located nodes transmitting all on the same channel at the same power and sending data to a random destination, Gupta and Kumar defined the throughput capacity as the average per node throughput feasible under suitable spatial and temporal scheduling schemes. With the protocol model, they were able to show that a throughput of $\Theta(1/\sqrt{n \log n})$ bps is achievable. In addition to this rough approximation, the physical model together with a constant $\gamma^*_{i,j}$ allows to find tighter bounds for specific scenarios.

While adequate for proving theoretical bounds for the network capacity, both models introduced by [5] are too complex for computing the throughput of a concrete network instance. The study of Li et al. [6] therefore considers 802.11 protocol overhead and localized traffic patterns which are, according to the authors, more likely to occur in large mobile ad hoc networks (MANETs) than completely random traffic flows. In a simulation study, the radio channel was abstracted by assuming that all nodes within a transmission range of 250 m and an interference range of 550 m are able to communicate or interfere respectively. Using these settings, the authors pointed out the inherent unfairness problems of the 802.11 channel access mechanism, but were also able to show that large MANETs with localized traffic patterns guarantee a per node throughput close to the theoretical optimum proved in [5].

An analysis for wireless mesh networks was presented by Jun and Sichitiu [7]. The authors argued that in WMNs, the traffic of all nodes has to be routed through the same Internet gateway, the per flow throughput is thus in the order of $O(1/a)$, where a is the average number of nodes per gateway node. For an exact throughput calculation, the concept of *collision domains* was used. The collision domain of link (i, j), $\mathcal{D}_{i,j}$, contains (i, j) and all other links which must be inactive for a successful transmission on (i, j). In [7], collision domains are defined on MAC layer to be either asymmetric if CSMA/CA, or symmetric if RTS/CTS is used.

We visualize this concept by analyzing the symmetric collision domain of link (4,3) in the example topology shown in Fig. 1. $(a, b) \in \mathcal{L}$ is a member of the symmetric $\mathcal{D}_{i,j}$ if either the link (a, j) or (i, b) exist, where $\forall_{i\in\mathcal{N}}(i, i) \in \mathcal{L}$. For sakes of simplicity, we consider only *active* links which are used for routing purposes. In the example, the links carrying the flows 2, 3 and 4 from the gateways 1 and 5 to the nodes, i.e. (1,2), (5,4), (4,3) are active and depicted by solid lines. The links (2,3), (3,2), (2,1), (3,4) and (4,5) are called *passive*, as they are not used for routing purposes. The symmetric collision domain of (4,3) is computed as $\mathcal{D}_{4,3} = \{(4, 3), (5, 4), (1, 2)\}$. Note that (1,2) is not part of the asymmetric $\mathcal{D}_{4,3}$.

Fig. 1. Simple example network

To determine the *nominal throughput* in a mesh network with homogeneous nodes, Jun and Sichitiu calculated the *nominal load* of the collision domains which corresponds to the number of transmissions taking place in a collision domain $\mathcal{D}$ [7]. A transmission is defined as one hop, a flow crossing $\mathcal{D}$ takes on a link of $\mathcal{D}$. The nominal load of $\mathcal{D}$, $m_{\mathcal{D}}$, is computed as the sum of all transmissions on all links of $\mathcal{D}$. If all links have the same rate R, the capacity of the whole collision domain is also R and the throughput of any flow traversing at least one link of $\mathcal{D}$ is $R/m_{\mathcal{D}}$. We use the previously discussed symmetric collision domain $\mathcal{D}_{4,3}$ from Fig. 1 for an illustration. In $\mathcal{D}_{4,3}$, flow 3 causes two transmissions, flow 4 and 2 cause each only one transmission. This results in a nominal load of $m_{\mathcal{D}_{4,3}} = 4$ and a maximal achievable *nominal* per flow throughput $\tau_n(2) = \tau_n(3) = \tau_n(4) = R/4$.

Computing the max-min fair per flow throughputs in a WMN is more complex than in this simple example. For the case of a mesh network with only one Internet gateway, the algorithm proposed by Aoun and Boutaba [8] is suitable. It uses the concept of *bottleneck collision domain* which is the collision domain with the highest nominal load and the lowest throughput. Starting with the collision domain containing the links to the gateway, the algorithm iteratively allocates the maximal achievable throughput to all flows traversing the bottleneck collision domain.

In [8], it was already suggested, that the *effective load* instead of the nominal load should be used for identifying the network bottleneck. The effective load of a collision domain is smaller or equal than its nominal load, as it accounts for the possibility, that two links in a collision domain can be used in parallel. In the example topology of Fig. 1 with symmetric collision domains, the transmissions on links (1,2) and (5,4) can not take place simultaneous to the transmissions on link (4,3), but they can take place in parallel to each other. The transmissions from 4 to 3 could thus be scheduled in a third of the time, and for two thirds of the time the transmissions from 1 to 2 and from 5 to 4 can take place in parallel. The latter one consisting of one half with destination 4 and one half with destination 3, thus resulting in the *effective* throughput $\tau_e(3) = \tau_e(4) = 1/3R$ and $\tau_e(2) = 2/3R$.

Aoun and Boutaba did not indicate how to compute the effective load for mesh networks. In [2] we therefore closed this gap and used the results of Huang et al. [9] in order to compute the max-min fair capacity of WMNs using both the effective and the nominal load. To compute the effective load of the links, we determine which transmissions may be scheduled at the same time by computing cliques in the so called *contention graph* $\mathcal{G_C} = (\mathcal{L}, \mathcal{K})$. The vertices of $\mathcal{G_C}$ are the active links between the mesh nodes. An edge $k \in \mathcal{K}$ between two links $j, l \in \mathcal{L}$ exists, if the two links are *contending*, i.e. may not be used in parallel [9]. We define the *clique corpus* $\Omega_\mathcal{C}$ of $\mathcal{G_C}$ as the set of all cliques of $\mathcal{G_C}$ which are not subset of another clique. Out of each clique $\mathcal{C} \in \Omega_\mathcal{C}$ one link may be used at a time. In Fig. 2, the contention graph for the topology of Fig. 1 is shown. The corresponding (active) clique corpus is given by $\Omega_\mathcal{C} = \{\{(1,2),(4,3)\},\{(4,3),(5,4)\}\}$.

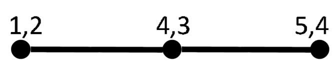


Fig. 2. Contention graph for network from Fig. 1

The effective load of a clique is computed as the sum of all flows traversing links of this clique [2]. We accounted for heterogeneous link rates by computing the effective and nominal loads as the sum of the carried traffic flows weighted by the corresponding link rates. Two max-min fair rate allocation algorithms emerged from this and other extensions: The Nominal Load Based Algorithm (NLBA) is an enhancement of the approach presented in [8]. It iteratively allocates the maximal feasible rate to the flows traversing the bottleneck collision domain. The Effective Load Based Algorithm (ELBA) iteratively allocates the maximal feasible rate to the flows traversing the bottleneck clique in the contention graph.

In [2], we furthermore considered adaptive modulation and coding. We studied topologies where for each link (i,j) the modulation and coding scheme k with an SINR requirement $\gamma_k^* =: \gamma_{i,j}^*$ that is just smaller than the link's *signal to noise ratio* (SNR) $\gamma'_{i,j} = R_{i,j}/N$ is used:

$$k = \arg\max\{l : \gamma_l^* \leq \gamma'_{i,j}\}. \quad (2)$$

If the SNR $\gamma'_{i,j}$ is smaller than the SINR requirement of the most robust MCS, i and j can not communicate, and thus, (i,j) does not exist. Otherwise, the link exists and its data rate $r_{i,j}$ is set to the data rate of the MCS resulting from Eq. (2).

B. Our contribution

For a thorough study of the effects of MCS choice on end-to-end throughput in WMNs, we use results presented in the last section and additionally introduce an MCS and interference aware collision domain definition which makes this abstraction more realistic. We also present a methodology for studying the effects of a conservative MCS selection strategy, which assigns more robust MCSs to the links than necessary. Additionally, we propose abstractions for different routing paradigms in order to examine the sensitivity of capacity results to routing paradigms.

1) A more realistic collision domain definition: Hamida et al. [10] identified the oversimplified physical layer modeling as main responsible for unrealistic simulation results. They examined the impact of relaxing different simplifying assumptions and found that a correct interference model is the most important factor for producing realistic simulation results.

Interference modeling is often done by using collision domains. Both the symmetric and asymmetric collision domain definitions introduced in the last section abstract the network to a graph and do not consider interferences from non-neighboring nodes. This reduces the computational complexity, but introduces an error in comparison to reality. In contrast, Dousse et al. [11] considered all nodes of the network as potential interferers. To model the efficiency of different MAC protocols, Eq. (1) together with a constant SINR threshold was modified by multiplying the interference $I_{i,j}$ by a weight coefficient $0 \leq \delta \leq 1$. The authors used this model for proving that under the worst case assumption of all nodes always transmitting, too large values of δ, i.e. bad MAC protocols, result in a not connected network. For the case of mesh networks with a CSMA/CA channel access and intermittent traffic, this model is computationally too expensive, as the value of δ is varying in time and space.

To find a channel abstraction which is both computational feasible and close to Eq. (1), we define the *single-interferer collision domain* $\mathcal{D}_{i,j}$, as the set of all links (a,b) which are using the same channel as (i,j), and where a's transmission causes a too high interference for j to decode a signal from i or i's transmission causes a too large interference for b to successfully receive a transmission from a:

$$\begin{aligned}\mathcal{D}_{i,j} = \quad & \{(a,b) : \tfrac{R_{i,j}}{N+R_{a,j}} < \gamma_{i,j}^* \ \vee \ \tfrac{R_{a,b}}{N+R_{i,b}} < \gamma_{a,b}^*\} \\ & \cap \{(a,b) : q_{a,b} = q_{i,j}\}. \quad (3)\end{aligned}$$

Note, that with Eq. (3), the collision domain and the contention graph change if another MCS, i.e. a different SINR threshold is used for (i,j). Using the collision domains, the set of edges $\mathcal{K}$ in the contention graph $\mathcal{G_C}$ which contains all tuples of contending links is defined by $(l_1, l_2) \in \mathcal{K} \Leftrightarrow l_1 \in \mathcal{D}_{l_2} \Leftrightarrow l_2 \in \mathcal{D}_{l_1}$.

Finding all single-interferer collision domains requires an effort which is proportional to the square of the number of active links. The contention graph can be derived from the set of collision domains with a linear effort, whereas finding all cliques in $\mathcal{G}_C$ is NP-complete. As we consider medium sized topologies, this is however still feasible. Our approach may be easily extended to n-interferer collision domains, where the interference of up to n nodes is considered, but requires a higher computational effort. In order to analyze a large number of scenarios, we do not include this extension in our analysis.

2) Link rate vs. spatial reuse: We explained earlier that the down-side of a higher data rate is a higher SINR requirement and hence a smaller spatial reuse, i.e. both a smaller maximal distance between transmitter and receiver and a larger area where no other node may transmit.

Numerous practical works aiming at maximizing the transmission speed while minimizing the transmission losses by adapting the modulation robustness to the channel conditions have been published, see e.g. [12]. In contrast, it has rarely been analytically studied if the network capacity can be increased by selecting smaller link rates. One of the few contributions addressing this problem is the work of Max et al. [13] which analyzes the effect adaptive modulation and coding scheme on the capacity of mesh networks. The authors assumed an optimal MAC protocol and a given routing algorithm and formulated the scheduling of concurrent transmitters, transmission durations and rates for minimizing the resource utilization while satisfying the traffic demands as an optimization problem. Using heuristics, the authors were able to prove that smaller link data rates allow for an increased number of concurrent transmissions and hence increase the system throughput, but gave no details on the characteristics of optimal schedules or MCS selection strategies.

In this work, we approach an optimal link rate assignment by systematically investigating the benefits of more conservative MCS selection strategies on the end-to-end flow throughput. Under "more conservative" we understand not to use the maximal feasible link rate, i.e. the MCS found by Eq. (2), but to use an MCS with an SINR requirement which is significantly smaller than the link SNR. This strategy results in links more robust against interferences and is modeled by selecting the MCS k as

$$k = \arg\max\{l : \gamma_l^* + \Delta \leq \gamma'_{i,j}\}. \quad (4)$$

The *protection threshold* Δ expresses to which degree link (i, j) is protected against interferences. The higher Δ, the more interference can be supported for a successful transmission from i to j and the corresponding collision domain and clique may become smaller.

Any $\Delta > 0$ may decrease the transmission error probability, but also the maximal link data rate. The spatial reuse, in contrast, may be increased, which is especially beneficial in dense topologies. If e.g. $\Delta = 5$ dB is used for the 802.11a setting we describe in Section III-A, the maximal distance of two nodes communicating using the least robust MCS, 64-QAM 3/4, decreases from 93.5 to 70.1 m. If on the other hand, for link (i, j) with $d_{i,j} = 112$ m the more robust 16-QAM 1/2 instead of 16-QAM 3/4 is used, a parallel transmission of node k with $d_{k,j} = 337$ m will not interfere the transmission of i to j.

3) The routing paradigm: The impact of the routing strategy on the network capacity is often neglected. Typical examples for this approach are [2,5,6,7] which either assumed an arbitrary or a shortest path topology to exist. However, Akyildiz et al. [1] pointed out that the throughput of wireless mesh networks may be increased using e.g. link quality performance metrics, which have not been used for previously proposed simpler MANET routing protocols like shortest path routing. Studies focusing on mesh networks, e.g. [8], therefore often mention these advanced metrics. An abstraction of a routing protocol taking into account the link rates has been used in the WMN study presented in [13].

We consider mesh networks with more than one Internet gateway. The routing topology connecting each mesh node to one gateway is thus a forest, consisting of several trees rooted in the gateway nodes. A comparative study of specific routing protocols is not the goal of this work, we are only interested, to what degree the effects of different link rate assignment policies are sensitive to the routing paradigm. For this purpose, we consider three different routing approaches.

As an abstraction for an ad-hoc routing protocol, we use *Minimum-hop routing (MH)*, where each node forwards its data to the neighboring node which is the closest to a gateway. A more sophisticated mesh routing protocol is modeled by *Maximum-capacity routing (MC)* which establishes a path between each gateway and the neighbor which is connected with the highest link rate. Until all nodes are connected, MC will establish a connection between an already connected node and its not connected highest rate neighbor. For comparison purposes, we abstract a chaotic topology by *Random routing (R)* which establishes routing trees rooted in the gateways, by iteratively connecting each node to a randomly chosen already connected neighbor.

III. Evaluation Framework

In this section we describe how the influence of more robust modulation and coding schemes is examined. In Section III-A we describe how we create sample network topologies where link rates are assigned using different values of Δ. Section III-B, contains a simple example which explains how each network instance is evaluated. In particular, we illustrate the influence of Δ on the link rates, collision domains, and cliques and hence on the maximal achievable throughputs calculated by NLBA and ELBA introduced in Section II-A.

A. Topology generation

As an abstraction for environmental constraints for the position of mesh nodes, we use two different grids for positioning non-gateway and gateway nodes. The positions of the $F = |\mathcal{F}|$ non-gateway nodes or access points are randomly chosen grid points of a grid with grid length l_1. For choosing the locations of the $G = |\mathcal{G}|$ gateway nodes, another grid with grid length l_2 is used. l_2 is by a factor $\sigma = F/G$ larger, i.e. there is a by σ smaller number of grid points for choosing the location of

TABLE I

IEEE 802.11A MCS, FER $\leq 1\%$, 1500 BYTE PAYLOAD, PATH GAIN EQ. (5)

MCS	raw rate	SNR threshold	maximal distance
BPSK 1/2	6 Mbps	3.5 dB	273.5 m
BPSK 3/4	9 Mbps	6.5 dB	230 m
QPSK 1/2	12 Mbps	6.6 dB	228 m
QPSK 3/4	18 Mbps	9.5 dB	193.67 m
16-QAM 1/2	24 Mbps	12.8 dB	160.2 m
16-QAM 3/4	36 Mbps	16.2 dB	131.7 m
64-QAM 2/3	48 Mbps	20.3 dB	103.8 m
64-QAM 3/4	54 Mbps	22.1 dB	93.5 m

the gateways. Furthermore we assume that the gateways are not the bottleneck of the Internet connection and set the speed of their Internet link to 100 Mbps.

To reduce the computational complexity, we assume that all nodes are transmitting on the same channel, i.e. $q_{i,j} = q$ for all $(i,j) \in \mathcal{L}$. For the parameterization of the channel model, we follow Goldsmith [14]. The ambient noise N is set to the product of the thermal noise spectral density -174 dBm/Hz and the system bandwidth 20 MHz, i.e. $N = -101$ dBm. To calculate the power, by which node j receives a signal transmitted by node i, we use the generic path gain model from [14]. In decibel scale for a reference distance of 10 m and a path loss exponent of 4, the power received by j is

$$R_{i,j} = T_x + g_{i,j} = T_x - 140.046 - 40 \cdot \log_{10}(d_{i,j}), \quad (5)$$

where $d_{i,j}$ is the distance of nodes i and j in km and all nodes use the same transmission power $T_x = 100$ mW $= 20$ dBm.

In Table I we compare the SINR requirements γ_k^* and the maximal feasible transmission distances which allow to meet an FER of 1% when an IP packet with 1500 Bytes payload is transmitted over an AWGN channel. Results for the modulation and coding schemes available in IEEE 802.11a are obtained by link level simulations. To decide, if link (i,j) is existing and which MCS is used, Eq. (4) together with the SNR computed using Eq. (5) is used. The SINR requirements for the different values of Δ are taken from Table I. If the SNR between two nodes is smaller than the SINR requirement of BPSK 1/2, they may not communicate directly, but their transmission may interfere with each other. The subset of active links is derived from the set of paths which emerge from generating the routing forest according to the three considered paradigms. As we are focusing on downlink data flows, the set of passive links contains all links in uplink direction.

B. An example

In Fig. 3 we show an exemplary mesh topology with 5 nodes whereof node 1 and 2 are gateways. In Fig. 3(a), the maximal feasible link rates are assigned, while the link rates resulting from a SNR protection threshold of $\Delta = 5$ dB are shown in Fig. 3(b). Note that links (5,3) and (3,5) do not meet the tightened requirements for BPSK 1/2 and are thus not existing for $\Delta = 5$ dB. MH routing results in flows 3 and 4 routed via gateway 2 and flow 5 via gateway 1. Active links are shown by solid lines, passive links by dashed lines.

1) Difference of NLBA and ELBA: When we introduced NLBA and ELBA in Section II-A, we already explained that

TABLE II

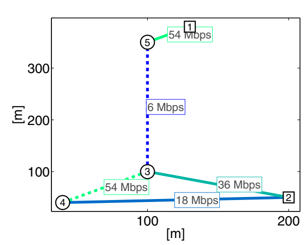
LINK RATES, COLLISION DOMAINS, CLIQUES AND MAX-MIN FAIR NOMINAL AND EFFECTIVE THROUGHPUTS FOR FIG. 3

	$\Delta = 0$ dB	$\Delta = 5$ dB
$r_{2,3}$	36 Mbps	24 Mbps
$r_{2,4}$	18 Mbps	12 Mbps
$r_{1,5}$	54 Mbps	54 Mbps
$\mathcal{D}_{2,3}$	{(2,3),(2,4),(1,5)}	{(2,3),(2,4)}
$\mathcal{D}_{2,4}$	{(2,3),(2,4)}	{(2,3),(2,4)}
$\mathcal{D}_{1,5}$	{(2,3),(1,5)}	{(1,5)}
$\Omega_{\mathcal{C}}$	{{(2,3),(2,4)},{(2,3),(1,5)}}	{{(2,3),(2,4)},{(1,5)}}
$\tau_n(3)$	9.82 Mbps	8 Mbps
$\tau_e(3)$	12 Mbps	8 Mbps
$\tau_n(4)$	13.09 Mbps	8 Mbps
$\tau_e(4)$	12 Mbps	8 Mbps
$\tau_n(5)$	39.27 Mbps	54 Mbps
$\tau_e(5)$	36 Mbps	54 Mbps

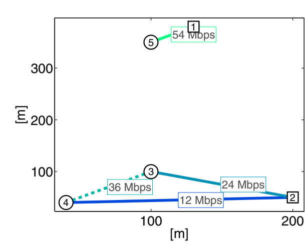
the effective load of a link is smaller or equal than its nominal load. We discuss the impact of the different load definitions on the per-flow throughputs in the following. Numerical results are summarized in Table II.

Let us focus on the case $\Delta = 0$ dB first. NLBA finds $\mathcal{D}_{2,3}$ as the bottleneck collision domain, as it contains all active links and its load is hence the largest. All flows, in the example flow 3, traversing link (2,3) are assigned the bottleneck rate, hence $\tau_n(3) = 9.82$ Mbps. For details on the computation of the bottleneck rate cf. [2]. After accounting for the time share, flow 3 requires in all other collision domains, this procedure is repeated resulting in $\tau_n(4) = 13.09$ Mbps and $\tau_n(5) = 39.72$ Mbps. For ELBA, the bottleneck clique is $\{(2,3),(2,4)\}$. Flows 3 and 4 traverse this clique and are fixed to the bottleneck rate $\tau_e(3) = \tau_e(4) = 12$ Mbps. Repeating this computation under the consideration of already occupied time shares results in $\tau_e(5) = 36$ Mbps.

This example demonstrates the effect of the different load definitions: The nominal load of each link depends on the number of flows crossing all links of its collision domain, resulting in large loads of collision domains with many links and hence very small throughputs for all flows crossing the corresponding link. As those flows are throttled, more capacity is available for flows not crossing bottlenecked links. Consequently, large rates may be assigned to those flows. The effective load concept used by ELBA allows for an increased spatial reuse. The cliques of the contention graph thus contain a smaller or equal number of links than the corresponding collision domains. Due to this effect, the flows which are traversing the bottleneck clique are assigned a rate which is larger or equal than the corresponding bottleneck collision



(a) $\Delta = 0$ dB



(b) $\Delta = 5$ dB

Fig. 3. Exemplary effects of link rate assignment

domain rate. In turn, the rates for the flows not crossing the bottleneck clique are smaller or equal, as the bottleneck flows are less throttled and hence occupy a larger percentage of the time. Consequently, ELBA produces more balanced end-to-end flow throughputs than NLBA.

2) Effect of more conservative MCS assignment: A more conservative MCS assignment strategy, represented by a positive value for Δ results in link rates smaller or equal to the maximal feasible rate for this link. Sometimes the link is even not existing as it is the case in our example. Our experiments show however that the throughput of a flow is not linearly related to the behavior of the link rates, sometimes the per-flow-throughput even increases with Δ. This is also the case in the example (cf. Table II): If a protection threshold is used for the MCS assignment, the maximal achievable throughput of node 5 increases to 54 Mbps under both load definitions, as the rate of link (1,5) remains unchanged at 54 Mbps, but a transmission of 1 is not disturbing the transmission of 2 to 3 any more. Obviously, flows 3 and 4 pay the price for the throughput increase of node 5. Both the nominal and effective throughput of node 3, 9.82 and 12 Mbps, and of node 4, 13.09 and 12 Mbps, are cut down to 8 Mbps. $\tau_n(3)$ is decreased to a smaller degree than $\tau_n(4)$ although the link rate (2,3) is reduced more heavily than the rate of link (2,4). This is due to the fact that node 3 does only need to share the available capacity with node 2 and no longer with node 5. The collision domain of link (2,4) remains however unchanged. This effect is not observable for the effective throughput, as the bottleneck clique $\{(2,3),(4,2)\}$ remains the same.

Analyzing the overall network throughput shows that an assignment of the maximal feasible link rates results in an average nominal throughput of 20.73 Mbps and thus slightly higher than the effective average throughput of 20 Mbps. If link rates are assigned with $\Delta = 5$ dB, both average throughputs increase to 23.33 Mbps. As we consider only three flows, the increase is rather modest, but already demonstrates the potential of the link rate assignment policy and could even be larger for other assignment policies. The next section will therefore be dedicated to a more thorough analysis of the effects of MCS choice.

IV. Numerical Results

In this section we use the previously introduced framework to investigate the influence of a more conservative MCS assignment on the max-min fair mesh network capacity. For this purpose, we study values of $\Delta = \{0, 1, \ldots, 10\}$ dB and consider mesh networks with 50 mesh nodes whereof 5 are gateways. Experiments with other parameterizations showed that the exact throughput results are topology-dependent, but that qualitative statements and trends are comparable.

To investigate the sensitivity of link rate adaption to external factors, we consider the influence of network density, the adaptation of the routing structure, and the connectivity of the topology. More precisely, we construct sample topologies with link rates assigned using different values of Δ for the following three scenarios:

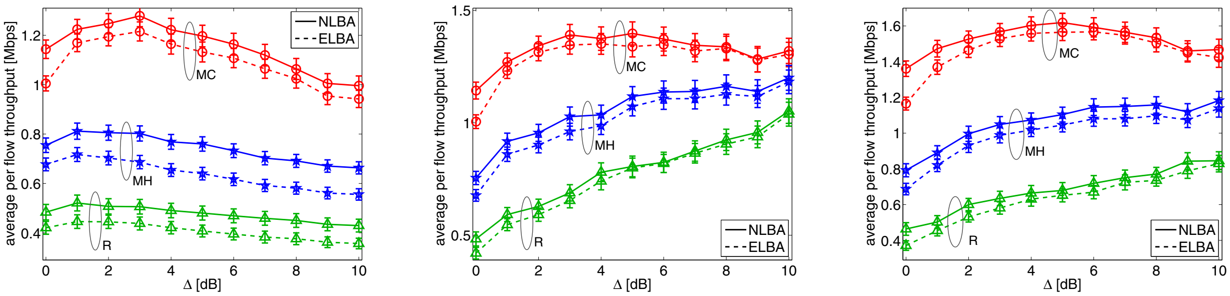
- *Scenario A*: Sparse network instances are generated using a grid length $l_1 = 20$ m. To keep the number of nodes constant for all sample networks, only topologies where all mesh nodes are connected to the gateways for $\Delta = 0$ dB are considered. The routing forest is determined for the maximal feasible link rates and not adapted for more conservative assignment strategies. No protection threshold is used for the most robust MCS in order to avoid isolated nodes for $\Delta > 0$ dB.
- *Scenario B*: Sparse networks with $l_1 = 20$ m which are connected for $\Delta = 0$ dB are considered. The protection threshold is used for all MCS which results in a smaller number of links for larger values of Δ and may cause nodes to loose their Internet connection. As the number of links is not constant the three routing forests are recomputed for each link rate assignment.
- *Scenario C*: Dense networks result from using $l_1 = 15$ m and considering only topologies where all nodes have an Internet connection for $\Delta = 10$ dB. As the protection threshold is used for all links, the number of links varies with different values of Δ and the routing forest is recomputed for each link rate assignment.

In the following, we first present general insights in the effect of link rate assignments on the average network throughput, before we analyze some details more closely.

A. The average network throughput

We use the average network throughput, i.e. the end-to-end flow throughput averaged over all end-to-end flows as metric for studying trends of more conservative link rate assignment strategies, i.e. increasing values of Δ. Fig. 4 shows the average network throughput for each considered experiment setup averaged over 200 different randomly generated network instances together with the corresponding 95% confidence intervals. The representation using three subfigures, line styles, and markers allows to compare the effects of more robust link rate assignment under different scenarios, load definitions, and routing paradigms respectively and illustrates the number of dimensions of the design space. Our first evaluation therefore addresses the different dimensions one by one.

Let us start with a comparison of the results obtained in the different scenarios. The only difference between the network instances created for Scenario A and B is the connectivity: using a more robust link rate assignment, link (i, j) may not exist in Scenario B, if its SNR its smaller than the requirements of BPSK 1/2 plus Δ. With higher values of Δ, some nodes even loose their Internet connectivity. As those unconnected nodes stop competing for the shared medium, other nodes can increase their throughput, and the overall system throughput is increased. We analyze this effect more closely in the next section. For now keep in mind, that in Scenario A, this link is kept operating with BPSK 1/2 and no node becomes unconnected. Recall that with increasing Δ the collision domains and cliques of the links get smaller, thus more concurrent transmissions are possible. In Scenario A, this effect holds however not for links with a SNR slightly



(a) Scenario A (b) Scenario B (c) Scenario C

Fig. 4. Average max-min fair end-to-end flow throughput

larger than the requirements of BPSK 1/2, as they can not use a more robust modulation and coding scheme. In Scenario A, the topologies are quite sparse, the percentage of these links is huge, and if the routing paradigm does not optimize the link rates (MH and R), this holds also for their percentage in the active set. More robust link rate assignment thus only affects a small number of links and is only slightly increasing the throughput. In Scenario B, those bottleneck links are removed from the set of links if Δ is increasing. The average throughput comparison of Fig. 4(a) with Fig. 4(b) and 4(c) illustrates thus a greater optimization potential in Scenarios B and C. Due to the dense node deployment, the proportion of short, i.e. high data rate links is larger, and the absolute throughput values shown in Fig. 4(c) tend also to be larger than the ones in Fig. 4(a) and 4(b).

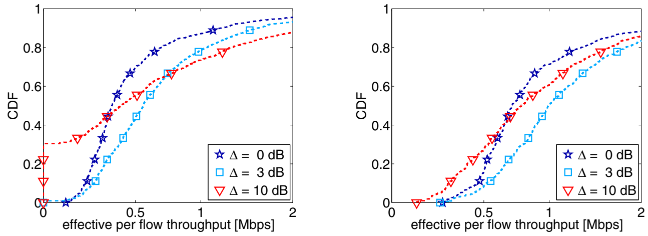
Now, we compare the effects of the different routing policies. Out of all considered routing paradigms, maximum-capacity routing achieves the highest throughput, as it maximizes the used link rates. Minimum-hop routing results in a higher average throughput than random routing, as the number of hops and thereby the number of potential bottleneck-links is minimized. Those performance differences are observable in all figures i.e. scenarios. The results shown in Fig. 4(b) and Fig. 4(c) illustrate the case where the routing forest is adapted to the link rate assignment. In this case, the random routing paradigm bears a greater optimization potential than the other paradigms. As it does not optimize the routing forest it benefits the most from the increasing probability that a randomly chosen link is contending with a smaller set of other links. This effect holds to a weaker degree for the more efficient MH routing and is only valid as long as the link rates do not become too small or too many nodes are disconnected. Observe that for each combination of topology type and routing paradigm a critical value of Δ exists which maximizes the average network throughput. A too large value of Δ greater than this critical value results in a high number of unconnected nodes and links operating at the lowest rate and hence decreases the average per-flow throughput. The figures illustrate, that in Scenarios B and C, this critical value is larger than 10 dB. MC routing in contrast optimizes the per-flow-throughput by choosing high-rate links, the critical value is thus much smaller for this paradigm. If the routing structure is not adapted to the new link rates (cf. Fig. 4(a)) MC benefits most from a conservative link rate assignment. MH and R use a higher percentage of low data rate links, which are not protected against interferences and thus profit less from the increased spatial reuse.

Next, consider the difference between the throughputs computed by NLBA and ELBA. As we already saw in Section III-B, NLBA assigns very small throughputs to flows crossing links with large collision domains and consequently very large rates to all non-bottlenecked flows. As we are considering a rather small number of links, this results in a higher average network throughput than the one computed by ELBA which assigns more balanced flow rates. Furthermore, the largest difference between nominal and effective average throughput is observable for $\Delta = 0$ dB in Scenario A as the collision domain sizes are more heterogeneous. The most important fact is however that both load definitions show the same overall effects of Δ's influence even if some of the computed flow throughputs are numerically different.

Finally, we concentrate on the influence of a more conservative MCS choice represented by increasing values of Δ. At first glance we see that this strategy is more beneficial if the routing forest is adapted to the new link rates, i.e. in Scenarios B and C. Especially for the non-throughput optimized minimum-hop and random routing paradigms, a high protection threshold enables significant performance gains in terms of average throughput. When comparing the routing paradigms, we already explained the existence of a critical value for Δ which limits the throughput increases shown in Fig. 4(b) and 4(c). We explore the reason for this more closely in the next section. For now, we would like to point out that using $\Delta > 0$ dB for MCS assignment is always suitable for increasing the network throughput, even if the routing is not optimized and low data rate bottleneck links are kept for connectivity reasons (cf. Fig. 4(a)).

B. A closer look on the per flow throughput

The average network throughput shown in Fig. 4 does not allow to judge how an increase is achieved. For a more thorough analysis, we therefore show the cumulative distribution functions (CDFs) for all effective end-to-end flow throughputs collected in the 200 considered topologies in Fig. 5. We use



(a) Scenario B, MH routing (b) Scenario C, MC routing

Fig. 5. Effective per flow throughput for Scenarios B and C

the effective load definition and compare only two scenarios, three values of Δ and two routing paradigms. This restriction still allows to illustrate our points while it increases the clarity of the presentation. For sakes of readability, we furthermore limit and logarithmically scale the x axis, as the flow rate distribution is highly variant. For $\Delta = 10$ dB, e.g. there is always one flow which achieves the highest throughput of 54 Mbps, as it is the only child of a gateways node and no other node is interfering its transmission.

We start with analyzing Fig. 5(a). Recall that in Scenario B, we use the protection threshold for all MCS. As the CDF of the per-flow throughputs for $\Delta = 10$ dB illustrates this may result in disconnected topologies, i.e. nodes with zero throughput. The exact value of the probability for a node to be disconnected depends on the network density but exists for all routing paradigms. Isolated nodes do not compete for time shares on the channel and all not isolated nodes are thus able to increase their throughput. Fig. 4(b) illustrates that this effect increases the average per-flow throughputs, as long as the number of isolated nodes is not too large. The CDF for $\Delta = 3$ dB demonstrates however also that medium values of Δ are suitable for a more balanced throughput increase.

In Fig. 5(b) we use Scenario C to address another detail. We explained earlier that for each routing paradigm and topology type a critical value of Δ exists which maximizes the average network throughput. The average throughputs for Scenario C shown in Fig. 4(c) illustrate that for MC routing this critical value is somewhere in $[4, 6]$ dB. The CDFs shown in Fig. 5(b) demonstrate that as long as Δ is below the critical value (cf. CDF for $\Delta = 3$ dB), the throughput of nearly all flows is increased in comparison to $\Delta = 0$ dB. The reason for this is that this moderate protection threshold causes only links with an SNR very close to the MCS threshold, thus contending with a large number of other links, to decrease their rate. This altruistic "behavior" of a small number of potential bottleneck links is advantageous for the entire network. If in contrast a value of Δ above the critical threshold is used (cf. CDF for $\Delta = 10$ dB), all links use a data rate one or several levels below the theoretical feasible fastest one. For the set of MCSs we considered, this may result in significant raw bit rate reductions (cf. Table I) which can not be compensated by increased channel access time and thus causes small flow rates. MC is using the highest proportion of high data rate links where the reductions are most severe and is thus more sensitive to this problem than the other routing paradigms.

V. Conclusion and Outlook

In this paper we investigated the potentials of link rate selection for increasing the max-min fair throughput of wireless mesh networks. For our analysis, we introduced a more realistic collision domain concept and the protection threshold Δ. This enabled us to investigate the advantages and disadvantages of more conservative MCS selection strategies. To make the study more thorough, we furthermore used two different analytical load definitions. Our methodology enabled us to show that using slightly more robust modulation and coding schemes than necessary is a suitable mechanism for increasing the overall network throughput. We demonstrated that for each network configuration an optimal protection threshold which increases the maximal achievable max-min fair throughput of all end-to-end flows can be found.

One critical point of our method is the use of the same protection threshold for all links. Our studies already showed that some links benefit to a higher degree from a protection than others. We therefore plan to extend our framework in order to allow for heterogeneous protection thresholds. Additionally, we will derive throughput maximizing protection thresholds taking into account traffic characteristics and network configurations while minimizing the number of isolated or badly connected nodes.

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