Distributed radio resource allocation for the downlink of multi-cell OFDMA radio systems

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Abstract—**This paper analyzes the problem of allocating power and subchannels in the downlink of a multi-cell, full-reuse OFDMA cellular system. We propose a distributed iterative algorithm, in which the allocation is performed independently in each cell, by maximizing the cell rate subject to a power constraint. The algorithm restricts the set of users that can be allocated on each subchannel, in order to satisfy a sufficient condition for the convergence to a stable equilibrium. Simulation results show that the proposed allocation and power control algorithm converges quickly and achieves a good spectral efficiency.**

I. Introduction

The main challenge for future wireless communication systems will be to provide wideband wireless access to a large number of subscribers, fulfilling at the same time strong requirements in terms of quality-of-service (QoS). Most of the candidate technologies of future generation broadband wireless networks employ a multiple access scheme based on the Orthogonal Frequency Division (OFDM) modulation [1], [2]. Provided that the system parameters are accurately dimensioned, OFDM transmissions are not affected by intersymbol interference (ISI) even in highly frequency-selective channels [3]. Moreover, OFDM can effectively exploit the channel frequency diversity by dynamically adapting power and modulation format on all subcarriers [4], [5]. In an OFDM based multiple-access (OFDMA) system a different subset of orthogonal subcarriers is allocated to each user (such subsets will be called *subchannels* in the following). If the transmitter possesses full knowledge of the channel state information (CSI) of each user, the overall spectral efficiency can be increased by allocating the subchannels according to certain optimality criteria, thus exploiting the so-called *multiuser diversity*. In recent years resource allocation has been envisaged as one of the most efficient techniques to increase the performance of single-cell multicarrier systems. Following the path open by the seminal article by Wong et al. [6], many resource allocation algorithms have been proposed to take advantage of both the frequency selective nature of the channel and the multi-user diversity. Most of the works in literature follow either the *margin adaptive* approach, formulating dynamic resource allocation with the goal of minimizing the transmitted power with a rate constraint for each user [7]-[9] or the *rate adaptive* approach aiming at maximizing the overall rate with a power constraint [10], [11]. In this latter case, the optimal solution for resource allocation in the downlink is often found as an application of the well-known waterfilling algorithm [12]. In particular, in [11] and [13] it is shown that OFDMA is the optimal multiple access scheme in a multiuser multicarrier downlink system. Furthermore, capacity is maximized by assigning each subchannel to the user with the maximum channel gain on it and distributing the power over subchannels using the water-filling solution with respect to the allocated channel gains. In [14] an iterative waterfilling solution is proposed for a multiple-access channel in a single-cell scenario where multiple uncoordinated transmitters send independent information to a common receiver.

All these works only consider allocation in a single cell. Because of its complexity, resource allocation in multi-cellular systems has not been fully studied yet and only few works tackle the problem [15]-[18]. In this paper we focus on the downlink of a multi-cellular system with universal frequency reuse. Allocation is performed in a distributed way, without resorting to a centralized allocator or to forms of explicit cooperation among cells. In this scenario inter-cell interference can have a tremendous impact on system performance. Each cell follows a greedy approach and allocates its resources aiming at maximizing its own objective function. Thus, because of the uncoordinated multi-access interference, the main problem of distributed resource allocation is the convergence of the proposed scheme and one of the primary goals of the allocation should be to avoid a disruptive interference among cells. In [19] we proposed a distributed resource allocation algorithm for a OFDMA system, which manages inter-cell interference by progressively reducing cell load until a stable traffic configuration is found. However, although the validity of the approach was shown through simulations, no analytic conditions for convergence were derived.

In this paper, we extend the scheme proposed in [11] and [13] to the multi-cellular case. Thus, a cell allocates its resources by assigning each subchannel to the user with the highest signal to noise ratio on that subchannel and the power is distributed using the water-filling solution with respect to the allocated channels. The proposed algorithm is iterative since the allocation in a particular cell interferes with the allocation in all other cells. The goal is the maximization of the total achieved data rate, subject to a per-cell power constraint. To our knowledge this is the first time that such approach is studied in an exact analytical framework in a multi-cellular system, while others [20]-[22] have studied the problem of distributed water-filling in the framework of ad-hoc networks. An important issue is whether the defined algorithm, in which each cell autonomously updates its subchannel assignments and transmission powers, converges to a stable equilibrium. A rapid convergence implies that the transmission power levels quickly stabilize, thus allowing an accurate prediction of the interference level on each subchannel. In this paper we present a sufficient condition for convergence.

The rest of the paper is organized as follows. Sections II describes the system model and Section III presents the allocation algorithms. In Section IV we derive the condition so that the proposed allocation schemes converge. Section V proposes a practical algorithm that takes advantage of the convergence condition found in the previous section to impose convergence also in those cases where convergence is not granted. In Section VI we present simulation results and, finally, in Section VII we draw the conclusions.

Boldface upper-case letters denote matrices, boldface lower-case letters denote column vectors, and italics denote scalars. The super-script $()^T$ denotes the transpose operation, $[A]_{m,n}$ denotes the (m-th, n-th) element of matrix A. The $[\mathbf{x}]_{\mathcal{P}_q}$ operator indicates the projection of $\mathbf{x}$ on the polyhedron $\mathcal{P}_q$. The $|\mathcal{A}|$ operator denotes the cardinality of the set $\mathcal{A}$, $\|\mathbf{X}\|_B$ indicates the block-norm of the matrix $\mathbf{X}$ and $\|\mathbf{x}\|_2$ indicates the 2-norm of the vector $\mathbf{x}$.

II. System model

We consider the downlink of a multi-cell infra-structured network with a complete reuse of the available frequency resources. There are K mobile terminals, distributed among Q cells; in each cell there is a Base Station (BS). The multiple access scheme is OFDMA: the overall frequency bandwidth is divided into M orthogonal subchannels, each of which can be assigned to one (and only one) user in the cell.

Each user feeds back its CSI, which includes the measured interference level and the path gains to the user's BS as well as those to the adjacent ones. On the basis of this information the BSs choose to which user each subchannel is to be assigned (*subchannel assignment*) and with what power (*power allocation*). We assume that the resource allocator of each BS possesses perfect CSI relative to all users in its cell. The allocator aims at maximizing the sum data rate achieved by users in the cell, subject to a power constraint. The optimization problem for cell q can thus be written as follows:

$$\max_{\boldsymbol{\rho},\mathbf{P}} \sum_{k\in\mathcal{U}(q)} \sum_{m=1}^{M} \rho(k,m) \log_2\left(1+\alpha_q\left(\mathbf{P}_{-q},k,m\right)P(k,m)\right) \tag{1a}$$

$$\rho(k,m)\in\{0,1\} \quad k=1,\dots,K \quad m=1,\dots,M \tag{1b}$$

$$P(k,m)\geq 0 \quad k=1,\dots,K \quad m=1,\dots,M \tag{1c}$$

$$\sum_{k\in\mathcal{U}(q)} \rho(k,m)\leq 1 \quad m=1,\dots,M \tag{1d}$$

$$\sum_{k\in\mathcal{U}(q)} \sum_{m} P(k,m) \leq P_{MAX}(q) \tag{1e}$$

where $\rho(k,m)$ takes the value 1 if the resource m is assigned to user k, and the value 0 otherwise; $P(k,m)$ is the power allocated to user k on subchannel m ($\mathbf{P}$ is thus a $K\times M$ matrix); $P_{MAX}(q)$ is the sum power bound for the cell q; $\mathcal{U}(q)$ is the set of users associated with the BS q; $\mathbf{P}_{-q}$ is the $(K-|\mathcal{U}(q)|)\times M$ matrix obtained from $\mathbf{P}$ by removing the rows associated with users in cell q and it represents the interference power configuration for cell q; $\alpha_q(\mathbf{P}_{-q},k,m)P(k,m)$ is the SINR at the receiver, and $\alpha_q(\mathbf{P}_{-q},k,m)$ is a positive factor that takes into account the propagation channel, the interference from other cells and the additive Gaussian noise:

$$\alpha_q(\mathbf{P}_{-q},k,m) = \frac{H_q(k,m)}{\sum_{l\neq q} H_l(k,m)\sum_{j\in\mathcal{U}(l)} P(j,m)+\sigma^2} = \frac{H_q(k,m)}{\sum_{l\neq q} H_l(k,m)p_l(m)+\sigma^2} \tag{2}$$

where $H_l(k,m)$ is the path gain from user k to the BS of cell l, on subchannel m, $p_l(m)=\sum_{j\in\mathcal{U}(l)} P(j,m)$ is the power at which BS l is currently transmitting on sub-carrier m, and σ^2 is the additive white Gaussian thermal noise power. In this paper we use Shannon's capacity as a measure of the achievable rate on a certain channel. In a practical system, where practical codes and modulation methods are used, the achievable rate can be computed by the same formula with the noise plus interference variance increased by a constant factor "SNR gap", which denotes the amount of extra coding gain needed to achieve Shannon capacity [23]. Without loss of generality, the SNR gap is assumed to be 0 dB for the rest of the paper, unless otherwise stated.

III. Subcarrier and power allocation

An accurate inspection of (1a) shows that the data rate on each subchannel is maximized by assigning that subchannel to the user experiencing the "best" link quality, i.e. to the user with the highest ratio between its path gain to the BS and the received interference [11], [13]. Subchannel m in cell q is thus assigned to the user k^* which maximizes $\alpha_q(\mathbf{P}_{-q},k^*,m)$, defined in (2). The allocation function $g_{q,m}:\mathbf{P}_{-q}\rightarrow\mathcal{U}(q)$ can be defined as follows:

$$g_{q,m}(\mathbf{P}_{-q}) = \arg\max_{k\in\mathcal{U}(q)} \frac{H_q(k,m)}{\sum_{l\neq q} H_l(k,m)p_l(m)+\sigma^2} = \arg\min_{k\in\mathcal{U}(q)} \left(\sum_{l\neq q} \tilde{H}_l(k,m)p_l(m)+\tilde{\sigma}^2(k,m)\right) \tag{3}$$

We used the notation:

$$\tilde{\sigma}^2(k,m) = \frac{\sigma^2}{H_{BS(k)}(k,m)} \tag{4}$$

$$\tilde{H}_l(k,m) = \frac{H_l(k,m)}{H_{BS(k)}(k,m)} \tag{5}$$

where $BS(k)$ denotes the BS to which terminal k is associated. The powers are then assigned to the users by water-filling, as follows:

$$p_q(m) = P(g_{q,m}(\mathbf{P}_{-q}), m) = \left[\mu_q - \frac{1}{\alpha_q(g_{q,m}(\mathbf{P}_{-q}), m)}\right]^+ = \left[\mu_q - \sum_{l \neq q} \tilde{H}_l(g_{q,m}(\mathbf{P}_{-q}), m) p_l(m) - \tilde{\sigma}^2(g_{q,m}(\mathbf{P}_{-q}), m)\right]^+ \quad (6)$$

where the water level μ_q has been chosen to satisfy the power constraint (1e).

IV. Convergence

Each BS performs the power and subchannel allocation algorithm independently of the other BSs. Thus, since the allocation in one cell interferes with the allocations in all other cells, the proposed algorithm is iterative. As for any iterative strategy a major concern is whether the proposed allocation converges to a *fixed point*, i.e. to an allocation that is stable in all cells. At the beginning of each new frame, all cells update their allocation simultaneously on the basis of the interference measured in the previous frame; if the allocation has reached a fixed point all cells maintain the previous allocation. Since we focus on a completely distributed multicellular scenario, in this paper we formulate a *simultaneous* update strategy rather than a sequential one. In fact, the latter scheme would require some form of coordination among cells.

In the following we will use the notation:

$$\tilde{\boldsymbol{\sigma}}_q^2(\mathbf{P}) = \left[\tilde{\sigma}^2(g_{q,1}(\mathbf{P}_{-q}), 1), \ldots, \tilde{\sigma}^2(g_{q,M}(\mathbf{P}_{-q}), M)\right]^T \quad (7)$$

$$\tilde{\boldsymbol{H}}_{l,q}(\mathbf{P}) = \operatorname{diag}\left(\tilde{H}_l(g_{q,1}(\mathbf{P}_{-q}), 1), \ldots, \tilde{H}_l(g_{q,M}(\mathbf{P}_{-q}), M)\right) \quad (8)$$

$$\mathbf{p}_q = [p_q(1), \ldots, p_q(M)] \quad (9)$$

It can be shown [24], [25] that waterfilling is equivalent to projecting the vector:

$$-\tilde{\boldsymbol{\sigma}}_q^2(\mathbf{P}) - \sum_{l \neq q} \tilde{\boldsymbol{H}}_{l,q}(\mathbf{P}) \mathbf{p}_l \quad (10)$$

on the polyhedron $\mathcal{P}_q$ defined as:

$$\mathcal{P}_q = \left\{ \mathbf{p}_q \in \mathcal{R}^M : \sum_{m=1}^{M} p_q(m) = P_{MAX}(q), 0 \leq p_q(m), \; m = 1, \ldots, M \right\} \quad (11)$$

Subcarrier allocation and water filling can thus be seen as a mapping $\mathbf{T}(\mathbf{P}) = \left[\mathbf{T}_1^T(\mathbf{P}), \ldots, \mathbf{T}_Q^T(\mathbf{P})\right]^T$; on each frame, cell q performs independently from the other cells the mapping:

$$\mathbf{T}_q(\mathbf{P}) = WF_q(\mathbf{P}_{-q}) = \left[-\tilde{\boldsymbol{\sigma}}_q^2(\mathbf{P}) - \sum_{l \neq q} \tilde{\boldsymbol{H}}_{l,q}(\mathbf{P}) \mathbf{p}_l\right]_{\mathcal{P}_q} \quad (12)$$

We omit the proof of the following Theorem due to lack of space:

Theorem 1: The mapping $\mathbf{T}(\mathbf{P})$ defined in (12) is a *block contraction* [26] under the block-maximum norm:

$$\|\mathbf{T}(\mathbf{P})\|_B = \max_q \|\mathbf{T}_q(\mathbf{P})\|_2 \quad (13)$$

if the following condition is satisfied:

$$\tilde{\beta} = \max_{q \in \{1, \ldots, Q\}} \sum_{l \neq q} \max_{k \in \mathcal{U}(q), m} \tilde{H}_l(k, m) < 1 \quad (14)$$

We can now immediately prove the following corollary:

Corollary 1: The user allocation and waterfilling power control algorithm defined by the mapping $\mathbf{T}(\mathbf{P})$ has a unique fixed point $\mathbf{P}^*$, and converges to it geometrically from any initial point $\mathbf{P}_0$, if condition (14) is satisfied.

Proof: As shown in Theorem 1, for $\tilde{\beta} < 1$ the mapping $\mathbf{T} : \mathbf{P} \rightarrow \mathbf{P}$ is a contraction, thus the allocation algorithm $\mathbf{P} := \mathbf{T}(\mathbf{P})$ is a *contracting iteration*, which is known to converge geometrically to a unique fixed point (see e.g. [26]). ■

We have found a sufficient condition for the allocation and power control algorithm based on rate maximization to converge to a stable solution. If we fix the allocation of subchannels, we can analogously prove that condition (14) is sufficient for the (waterfilling) power allocation algorithm to converge to a stable allocation.

V. Practical power and user allocation algorithms

In the previous section we have found a sufficient condition for the convergence of the proposed algorithm that combines subchannel allocation and waterfilling in a multicellular OFDMA scenario. This section presents the practical implementation of the proposed algorithm introducing a subchannel removal algorithm that forces the convergence condition (14) on the set of available channels in each cell. Moreover, we also introduce the uniform power allocation algorithm, whose performance will be used as a comparison term in the results section.

A. Waterfilling allocation (WFA)

The pseudocode of the basic WaterFilling Allocation algorithm (WFA), described in Section II, is presented in Algorithm 1. We use the same notation as in the previous Section; however, here we explicitly introduce the time dimension, represented by the index t of the current allocation interval (frame). The power level of cell q on subchannel m in frame t is denoted as $p_q(m, t)$, while $g_{q,m}(t)$, similarly to (3), denotes the user to which subchannel m is allocated. Thus the selection of the user with the highest SINR (line 5) is made by taking into account the interference level in the previous time interval.

B. Waterfilling and Subcarrier Removal Allocation (WSRA)

Note that there is no guarantee that the WFA algorithm converges to a stable allocation, unless the sufficient condition (14) is satisfied. It is possible, however, to guarantee convergence, by preventing certain users from being assigned to certain subchannels. We rewrite the convergence condition (14) for a particular cell q:

$$\sum_{l \neq q} \max_{k \in \mathcal{U}(q), m} \tilde{H}_l(k,m) < 1 \tag{15}$$

We now modify the allocation algorithm by forcing the BS of cell q to choose the user to be allocated on subchannel m from a set $\mathcal{W}(q,m) \subseteq \mathcal{U}(q)$. The allocation rule is still the same as in (3), but the permissible set $\mathcal{W}(q,m)$ has been reduced so as to satisfy the convergence condition, i.e.:

$$\sum_{l \neq q} \max_{m \in \{1,\ldots,M\}, k \in \mathcal{W}(q,m)} \tilde{H}_l(k,m) < 1 \tag{16}$$

Note that choosing a set $\mathcal{W}(q,m)$ so that (16) holds may result in some subchannels to be completely unused.

Restricting the set of users which can be allocated on each subchannel does not change the nature of the allocation algorithm, thus the results found in Section IV still apply. In fact, not allowing a given user k to be allocated on a subchannel m is equivalent to replacing the path gain between the user and the BS with a fictitious very low value ϵ, ie $H_{BS(k)}(k,m) = \epsilon$. This path gain is chosen so as to prevent the allocation algorithm, which selects the best user, from choosing user k for subchannel m; if all the other users have also been switched off, user k may still be selected, but the high path loss ensures that waterfilling allocates no power to subchannel m.

A heuristic algorithm based on these principles (Waterfilling and Subcarrier Removal Allocation - WSRA) is presented as pseudocode in Algorithm 2. The allocation is performed by selecting for each subchannel the user with the highest SINR among those which satisfy condition (16), computed by taking account of the allocation made so far (line 19). When performing the allocation, subchannels are considered in the order given by the path gain of the best user on each subchannel (lines 10-13). After subchannels have been assigned, transmission power is allocated with waterfilling (lines 29-30). Note that the algorithm is distributed as the decision is taken independently in each cell. Moreover, a BS only has to take into account the values of the path gains between the users in its cell and the other BSs; these values can be measured by the terminals and then signalled to the BS.

C. Uniform power allocation

Finally we introduce a simple algorithm which allocates power uniformly among the subchannels (Uniform Power Allocation - UPA). In order to maximize the total achieved throughput, to each subchannel we allocate the user with the highest SINR, as was done in WFA. The pseudocode for UPA is shown in Algorithm 3.

VI. Application Example

The transmission capacity of wireless networks can be increased by reducing the distance between Base Stations, thus lowering the transmission power and increasing frequency reuse. For these reasons, there is considerable interest in femtocell technology [27]: low-power Femto-BSs are overlaid on the existing cellular network, and provide a high-speed wireless connection to subscribers within a small range (e.g., a house or a small office). They can be installed in locations where users are experiencing unsatisfactory signal reception and are unable to achieve high data rates. Interference management is one of the main technical challenges facing femtocell technology: Femto-BSs will be deployed by end consumers, thus making it difficult to perform centralized frequency planning and to have coordination among Femto-BSs and between Femto-BSs and macro BSs. Because of this, universal frequency reuse is an attractive solution, provided that the resulting intercell interference can be managed. Femtocells share their frequency band with macro BSs, and with nearby femtocells, thus making the interference scenario particularly challenging.

In the following we consider a femtocell scenario, characterized by small cell radius and low power budget. The system comprises $Q = 7$ femtocells of radius $R = 50$ m each. Channel attenuation is only due to path loss, proportional to the distance between the BS and the mobile user; the path loss exponent is $\alpha = 4$. We assume that the users are uniformly distributed within the cell and that each user's channel undergoes Rayleigh distributed fading with exponentially decaying power delay profile. The OFDM signal spans a $B = 10$ MHz bandwidth at a carrier frequency of $f_0 = 2.3$ GHz and is transmitted on $M = 64$ orthogonal sub-carriers. Each cell has the same number of users K_{cell} as the other cells, so there are $K = Q \cdot K_{cell}$ mobile stations in the system.

Note that the allocation algorithm presented in Section V aims at throughput maximization, with no regard for fairness. In order to achieve throughput fairness among the users in each cell, we adopt the credits-based packet scheduler (PS) presented in [28]. The time axis is organized into frames; in each frame, algorithm 2 allocates resources to a given set $\mathcal{U}$ of users eligible for allocation. This set is determined by the PS, and is dynamically modified with the goal of guaranteeing fairness among users in the long term. So as to allow the allocation to converge in the multicell system, the set $\mathcal{U}$ is kept unvaried for the duration of a *scheduling interval*, made up of 10 frames.

Fig. 1 shows the probability of convergence for the WSRA and WFA algorithms versus the number of users in a cell. The UPA always converges. For a small number of users a $\beta < 1$ so that (14) is true may well not exist but, as expected, WSRA always converges. Multi-user diversity has a positive effect on the convergence of WFA: as the number of users increases, so does the probability of finding a stable power allocation using WFA. Note that, as described in Section V-B, subchannels are not allocated if it is not possible to do so by respecting the convergence conditions (14). As the number of users grows, so does the probability of finding one such allocation. Fig. 2

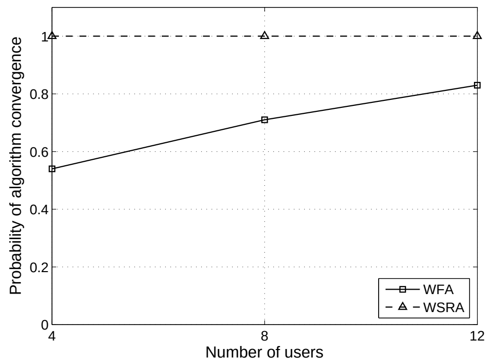

Fig. 1. Probability of convergence of WSRA and WFA vs number of users

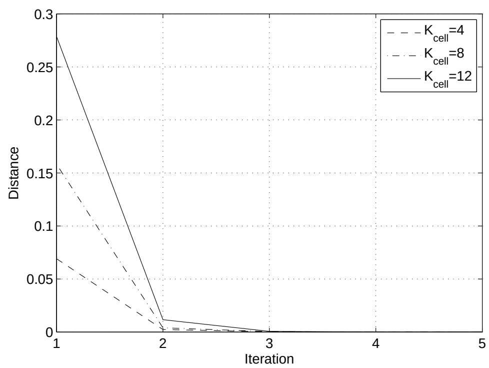

Fig. 2. Distance from the equilibrium vs time

shows the convergence of the WSRA algorithm, by plotting the distance[1] from the equilibrium allocation vs time. The figure shows that the algorithm converges very quickly.

Fig. 3 plots the average cell throughput normalized by the total bandwidth vs the transmission power in a system with $K_{cell} = 4$ users per cell. In the selected scenario WSRA clearly outperforms UPA. As expected the average throughput grows with the transmission power, but overall performance is limited by the inter-cell interference; in fact, a comparison with the results obtained considering the single-cell scenario, i.e. $Q = 1$, reveals that as the cell power increases the average throughput per cell becomes a small fraction of that achieved without interference. Fig. 4 plots the ratio of the throughput of the multi-cell (R_{MC}) scenario to that of the single-cell scenario (R_{SC}), for both UPA and WSRA.

[1]The distance of a set of power values $p^{\ell}(k,m)$ at the ℓ-th iteration from the equilibrium set of power values $p^{(eq)}(k,m)$ is defined as follows:

$$d(\ell) = \frac{\sum\limits_{k,m} \left| p^{(\ell)}(k,m) - p^{(eq)}(k,m) \right|^2}{\max_{k,m} \left| p^{(eq)}(k,m) \right|^2} \qquad (17)$$

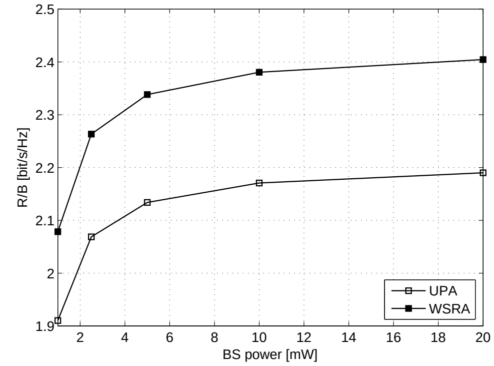

Fig. 3. Average throughput normalized by total bandwidth vs transmission power

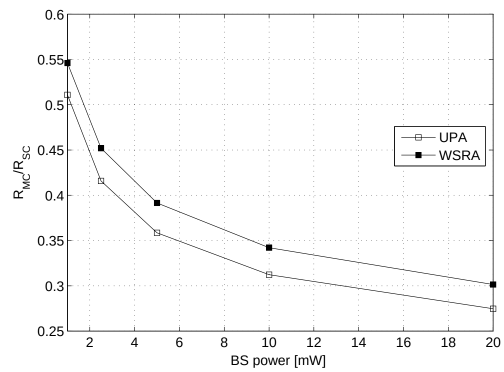

Fig. 4. Ratio of the average throughput for the single cell scenario to that of the multi-cell scenario, vs transmission power

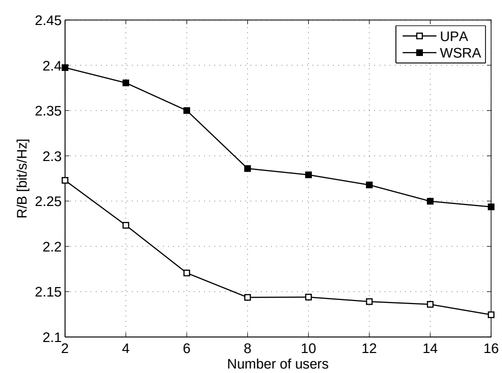

Fig. 5. Average throughput per cell vs number of users

Fig. 5 shows the average normalized throughput per cell versus the number of users. In this case the BS power constraint is set to $P_{MAX} = 10mW$. The average throughput remains stable as the number of users grows, since the benefits of multiuser diversity are offset by the negative impact of users on the cell border (the average distance from the BS of the most distant user increases with the number of users; since we are using a fair scheduler, such users have a significant impact on system performance).

VII. CONCLUSIONS

In this paper we have studied the problem of resource allocation and power adaptation in the downlink of a multicellular OFDMA system. The allocation problem has been formulated with the goal of maximizing the rate of each cell subject to a power constraint in a full frequency reuse scenario without any central controller. In such a setting inter-cell interference is the disturbance that mostly affects system performance. The optimal solution is first to allocate each subchannel to the user with the highest signal to noise plus interference ratio, and then to distribute the power using the water-filling policy. Since the allocation in a cell interferes with the allocations in all other cells, the proposed algorithm is iterative. We have found a sufficient condition for the convergence of this algorithm to a stable equilibrium. Therefore, we propose a heuristic algorithm that enforces the stability condition by removing the subchannels with the worst channel gains. Simulation results show that the proposed allocation and power control algorithm converges in just a few iterations. To avoid the computational burden in calculating the water-filling level in the proposed transmit power adaptation method, we also consider a uniform power allocation scheme. Results show that the heuristic based on the waterfilling policy significantly outperforms the algorithm based on uniform power allocation.

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Algorithm 1 WaterFilling Allocation

$\mathcal{U}(q)$ is the set of users in cell q
t is the frame index
for $q = 1$ to Q **do**
  for $m = 1$ to M **do**
    $g_{q,m}(t) \leftarrow \arg\min_{k\in\mathcal{U}(q)} \left(\sum_{l\neq q} \tilde{H}_l(k,m)p_l(m,t-1) + \tilde{\sigma}^2(k,m)\right)$
  end for
  waterfilling: compute $\mu_q(t)$, $p_q(m,t)$ so that $\sum_{m=1}^{M} p_q(m,t) = P_{MAX}(q)$
  $p_q(m,t) = \left[\mu_q - \frac{1}{\alpha_q(g_{q,m}(t))}\right]^+$
end for

Algorithm 2 Waterfilling and Subcarrier Removal Allocation (WSRA), second version

$\mathcal{U}(q)$ is the set of users in cell q
$\mathcal{W}(q,m)$ is the set of users which can be allocated to subchannel m in cell q
$\mathcal{M}(q)$ is the set of available subchannels in cell q
t is the time index
$StableAllocation \leftarrow 0$
repeat
  for $q = 1$ to Q **do**
    $\mathcal{M}(q) \leftarrow \varnothing$
    $\widehat{\mathcal{M}}(q) \leftarrow \{1,\ldots,M\}$
    for $m = 1$ to M **do**
      $g_{q,m}(t) \leftarrow 0$
      $\widehat{H}_q(m) = \arg\max_{k\in\mathcal{U}(q)} H_q(k,m)$
    end for
    while $\widehat{\mathcal{M}}(q) \neq \varnothing$ **do**
      $m \leftarrow \arg\max_{m\in\widehat{\mathcal{M}}(q)} \widehat{H}_q(m)$
      $\mathcal{W}(q,m) \leftarrow \mathcal{U}(q)$
      repeat
        $g_{q,m}(t) \leftarrow \arg\min_{k\in\mathcal{W}(q,m)} \left(\sum_{l\neq q} \tilde{H}_l(k,m)p_l(m,t-1) + \tilde{\sigma}^2(k,m)\right)$
        $StabilityCondition \leftarrow \left(\sum_{l\neq q} \max_{m^*\in\mathcal{M}(q)\cup\{m\}} \tilde{H}_l(g_{q,m^*}(t),m^*) < 1\right)$
        if $StabilityCondition = 0$ **then**
          $\mathcal{W}(q,m) \leftarrow \mathcal{W}(q,m) \setminus \{g_{q,m}(t)\}$
        end if
      until $StabilityCondition \vee (|\mathcal{W}(q,m)| = 0)$
      $\widehat{\mathcal{M}}(q) \leftarrow \widehat{\mathcal{M}}(q) \setminus \{m\}$
      if $StabilityCondition$ **then**
        $\mathcal{M}(q) \leftarrow \mathcal{M}(q) \cup \{m\}$
      end if
    end while
    waterfilling: compute $\mu_q(t)$, $p_q(m,t)$ so that $\sum_{m\in\mathcal{M}(q)} p_q(m,t) = P_{MAX}(q)$
    $p_q(m,t) = \left[\mu_q - \frac{1}{\alpha_q(g_{q,m}(t))}\right]^+$
  end for
  if $(\mathbf{g}(t) = \mathbf{g}(t-1)) \wedge (\mathbf{p}(t) = \mathbf{p}(t-1))$ **then**
    $StableAllocation \leftarrow 1$
  end if
  $t \leftarrow t+1$
until $StableAllocation$

Algorithm 3 Uniform Power Allocation

$\mathcal{U}(q)$ is the set of users in cell q
t is the frame index
for $q = 1$ to Q **do**
  for $m = 1$ to M **do**
    $g_{q,m}(t) \leftarrow \arg\min_{k\in\mathcal{U}(q)} \left(\sum_{l\neq q} \tilde{H}_l(k,m)p_l(m,t-1) + \tilde{\sigma}^2(k,m)\right)$
    uniform power allocation: $p_q(m,t) \leftarrow P_{MAX}(q)/M$
  end for
end for