Scheduling and Capacity Estimation in LTE

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Abstract— **Due to the variation of radio condition in LTE the obtainable bitrate for active users will vary. The two most important factors for the radio conditions are fading and pathloss. By including both fast fading and shadowing and attenuation due to distance we have developed analytical models to investigate obtainable bitrates for the basic resource unit in LTE. In addition we estimate the total cell throughput/capacity by taking scheduling into account. Particularly, the cell throughput is investigated for three types of scheduling algorithms; Round Robin, Proportional Fair and Max SINR (signal-to-interference-plus-noise ratio) were also fairness among users is part of the analysis. In addition models for cell throughput/capacity for a mixture of Guaranteed Bit Rate (GBR) and Non-GBR greedy users are derived.**

Numerical examples show that multi-user gain is large for the Max-SINR algorithm, but also the Proportional Fair algorithm gives relative large gain relative to plain Round Robin. The Max-SINR has the weakness that it is highly unfair when it comes to capacity distribution among users. Further, the model visualize that use of GBR with high rates may cause problems in LTE due to the high demand for radio resources for users with low SINR, i.e. at cell edge. Hence, for GBR sources the allowed guaranteed rate should be limited.

Index Terms— **LTE, scheduling, capacity estimation, GBR.**

I. Introduction

The LTE (Long Term Evolution) standardized by 3GPP is becoming the most important radio access technique for providing mobile broadband to the mass market. The introduction of LTE will bring significant enhancements compared to HSPA (High Speed Packet Access) in terms of spectrum efficiency, peak data rate and latency. Important features of LTE are MIMO (Multiple Input Multiple Output), higher order modulation for uplink and downlink, improvements of layer 2 protocols, and continuous packet connectivity [1].

While HSPA mainly is optimized data transport, leaving the voice services for the legacy CS (Circuit Switched) domain, LTE is intended to carry both real time services like VoIP in addition to traditional data services. The mixture of both real time and non real time traffic in a single access network requires specific attention, where the main goal is to maximize cell throughput while maintaining QoS and fairness for both users and services. Therefore radio resource management will be a key part of future modern wireless networks.

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For bandwidth efficiency, where only a single user is considered, the scheduling is without any significance. This is not the case when several users are competing for the available radio resources. The scheduling algorithms studied in this paper are those only depending on the radio conditions, i.e. opportunistic scheduling where the scheduled user is determined by a metrics depending on the SINR (signal-to-interference-plus-noise ratio). The most commonly known opportunistic scheduling algorithms are of this type like PF (Proportional Fair), RR (Round Robin) and Max-SINR. The methodology developed will, however, apply for general scheduling algorithms where the scheduling metrics for a user is a (known) function of SINR [2]. Also the multi user gain (i.e. the relative increase in cell throughput) due to the scheduling is of main interest. The proposed models demonstrate the magnitude of this gain. As for Max SINR algorithm this gain is expected to be huge, however, the gain comes always at a cost of fairness among users. Therefore fairness has to be taken into account when evaluating the performance of scheduling algorithms.

It is likely that LTE will carry both real time traffic and elastic traffic. We also analyze scenarios where a cell is loaded by two traffic types; high priority GBR (Guaranteed Bit Rate) traffic that requires a fixed data-rate and low priority (greedy) data sources that always consume the leftover capacity not used by the GBR sources. This is actual a very realistic traffic scenario for future LTE networks where we will have a mix of both real time traffic like VoIP and data traffic.

The remainder of this paper is organized as follows. In section II the basic radio model is given and models for bandwidth efficiency are discussed. Section III gives an outline of the multiuser case where resource allocation and scheduling is taking into account. Some numerical examples are discussed in section IV and in section V some conclusions are given.

II. Spectrum Efficiency

A. Obtainable bitrate per symbol rate as function of SINR

For LTE the obtainable bitrate per symbol rate will depend on the SINR (for both up-and downlink). The actual radio signal quality is signaled over the radio interface by the so-called CQI (Channel Quality Indicator) index ranging from 1 to 15. Based on the CQI value the coding rate is determined on basis of the modulation QPSK, 16QAM, 64QAM and the amount of redundancy included. The corresponding bitrate per bandwidth is standardized by 3GPP [3] and is shown in Table 1 below. For analytical modeling the actual CQI measurement

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procedures are difficult to incorporate into the analysis due to time lags, i.e. the signaled CQI is based measurements taken in earlier TTIs (Transmission Time Interval). To simplify the analyses, we assume that this time lag is set to zero and that the CQI is given as a function of the momentary SINR, i.e. we simply take CQI=CQI(SINR). This approximation is justified if the time variation in SINR is significantly slower than the length of a TTI interval. By applying the CQI table found in [3] we get the obtainable bitrate per bandwidth as function of the SINR as the step function:

$$B = f\ c_j \quad for \quad SINR \in \left[g_j, g_{j+1}\right);\ \ j = 0,1,...,15 \qquad (1)$$

where f is the bandwidth of the channel, c_j is the efficiency for CQI index j (as given by Table 1) and $\left[g_j, g_{j+1}\right)$ are the corresponding intervals of SINR values. (We also define $c_0 = 0$, $g_0 = 0$ and $g_{16} = \infty$.)

TABLE 1 CQI TABLE.

CQI index	modulation	code rate x 1024	efficiency
0	out of range		
1	QPSK	78	0.1523
2	QPSK	120	0.2344
3	QPSK	193	0.3770
4	QPSK	308	0.6016
5	QPSK	449	0.8770
6	QPSK	602	1.1758
7	16QAM	378	1.4766
8	16QAM	490	1.9141
9	16QAM	616	2.4063
10	64QAM	466	2.7305
11	64QAM	567	3.3223
12	64QAM	666	3.9023
13	64QAM	772	4.5234
14	64QAM	873	5.1152
15	64QAM	948	5.5547

To fully describe the bitrate function above we also have to also specify the intervals $\left[g_j, g_{j+1}\right)$. Several simulation studies e.g. [4] suggest that there is a linear relation between the CQI index and the actual SINR limits in [dB]. In the numerical examples we therefore take $SINR_j[dB] = 10\log_{10} g_j = aj + b$ or $g_j = 10^{\frac{aj+b}{10}}$ for some constants a and b. It is also argued that the actual range of the SINR limits in [dB] is determined by the following (end point) observations: SINR[dB]=-6 corresponds to CQI=1, while SINR[dB]=20 corresponds to CQI=15. Hence, with this assumption we then have $-6 = a + b$ and $20 = 15a + b$ or $a = 13/7$ and $b = -55/7$.

B. Radio channel models

Generally SINR for a user will be the ratio of the received signal strength divided by the corresponding noise. The received signal strength is the product of the power P_w times path loss G and divided by the noise and interference component N_f, i.e. $SINR = \frac{P_w G}{N_f}$. Now the path loss G will typical be a stochastic variable depending on physical characteristics such as rapid and slow fading, but will also have a component that are dependent on distance (and possible also the sending frequency). We first consider variations that are slowly varying over time intervals that are relative long compared with the TTIs (Transmission Time Intervals). Then the path loss is usually given in [dB] on the form:

$$G = 10^{L/10} \text{ with } L = C - A\log_{10}(r) + X_t \qquad (2)$$

where C and A are constants, A typical in the range 20-40, and X_t a slowly variation normal stochastic process with zero mean representing the shadowing effects (i.e. slow fading). The other important component determining SINR is the noise and interference. It is common to split the noise and interference power into two terms: $N_f = N_{\text{int}} + N_{ext}$ where N_{int} is the internal (or own-cell) noise and interference power and N_{ext} is the external (or other-cell) interference. In a CDMA (Code Division Multiple Access) network, the lack of orthogonality induces own-cell interference. In OFDMA (Orthogonal Frequency Division Multiple Access) networks, however, there is a perfect orthogonality between frequencies and therefore the only contribution to N_{int} is the thermal noise at the receiver. The interference from other cells depends on the location of surrounding base stations and will typically be largest at cell edges. To make the analysis mathematical traceable we shall assume that the external noise and interference is constant throughout the cell or negligible, i.e. we assume the noise N_f to be constant throughout the cell.

Hence, with the assumptions stated above, we may write SINR on the form $S_t/h(r,\lambda)$ where S_t represent the stochastic variations which we assume to be distance independent capturing the slowly varying fading, and $h(r,\lambda)$ represent the distance dependant attenuation (which we also may to depend on the sending frequency). Most commonly used channel models as described above have attenuation that follows a power law, i.e. we chose to take $h(r,\lambda)$ on the form

$$h(r,\lambda) = h(\lambda) r^{\alpha} \qquad (3)$$

where $\alpha = A/10$ is typical in the range 2-4 and $h(\lambda) = \frac{N}{P_w} 10^{-C/10} = 10^{Z/10}$ with $Z = 10\log_{10}(N) - 10\log_{10}(P_w) - C$ given [dB], where we also indicate that $h(\lambda)$ may depend of the (sending) frequency. With the description above the stochastic variable $S_t = 10^{X_t/10} = e^{S_t^\bullet}$ with $S_t^\bullet = \frac{\ln 10}{10} X_t$, and hence S_t is a lognormal process with $E[S_t^\bullet] = 0$ and $\sigma = \frac{\ln 10}{10}\sigma(X_t)$ where $\sigma(X_t)$ is the standard deviation (given in [dB]) for the normal process X_t. With these assumptions we have the Probability Density Function (CDF) and Complementary Distribution Function (PDF) of S_t as:

$$s_{\ln}(x) = \frac{1}{\sqrt{2\pi}\sigma x} e^{-\frac{(\ln x)^2}{2\sigma^2}} \text{ and } \widetilde{S}_{\ln}(x) = \frac{1}{2} erfc\left(\frac{\ln x}{\sigma\sqrt{2}}\right) \qquad (4)$$

where $erfc(y)=\frac{2}{\sqrt{\pi}}\int_{x=y}^{\infty}e^{-x^2}dx$ is the complementary error function.

C. Including fast fading

There are several models for fast fading in the literature like Rician fading and Rayleigh fading [5]. In this paper we restrict ourselves to the latter mainly because it describes the fast fading variations by a simple negative exponential distribution. It is possible to include fast fading into the description above. To do so we assume that the fast fading effects are on a much more rapid time scale than the slow fading. We therefore assume that the slow fading is constant during the rapid fading variations. Hence, condition on the slow fading to be y then for a Rayleigh faded channel the SINR will be exponentially distributed with mean $y/h(r,\lambda)$. We may therefore take SINR as $S_t/h(r,\lambda)$ where $S_t = X_{\ln}X_e$ is the product of a Log-normal and a negative exponential distributed variables. The corresponding distribution often called Suzuki distribution have PDF and CDF given by the integrals:

$$s_{su}(x)=\int_{t=0}^{\infty}\frac{1}{t}e^{-\frac{x}{t}}s_{\ln}(t)dt \text{ and } \tilde{S}_{su}(x)=\int_{t=0}^{\infty}e^{-\frac{x}{t}}s_{\ln}(t)dt \qquad (5)$$

where $s_{\ln}(t)$ is the lognormal PDF above by (4). Since $s_{\ln}(\frac{1}{t})=t^2 s_{\ln}(t)$ it is possible to express the integrals above in terms of the Laplace transform of the Log-normal distribution and therefore the CDF (and PDF) of the Suzuki distribution may be written as:

$\tilde{S}_{su}(x)=\hat{S}_{\ln}(x)$ and $s_{su}(x)=-\hat{S}'_{\ln}(x)$ where

$$\hat{S}_{\ln}(x)=\int_{t=0}^{\infty}e^{-xt}s_{\ln}(t)dt=\frac{1}{\sqrt{2\pi}\sigma}\int_{t=0}^{\infty}\frac{e^{-t-\frac{(\ln(t/x))^2}{2\sigma^2}}}{t}dt \qquad (6)$$

is the Laplace transform of the Log-normal distribution. If we define the truncated transform:

$$\tilde{S}_{su}(x,T)=\frac{1}{x}\int_{t=0}^{T}e^{-t}s_{\ln}(t/x)dt=\frac{1}{\sqrt{2\pi}\sigma}\int_{t=0}^{T}\frac{e^{-t-\frac{(\ln(t/x))^2}{2\sigma^2}}}{t}dt \qquad (7)$$

then $\tilde{S}_{su}(x)=\lim_{T\to\infty}\tilde{S}_{su}(x,T)$ and further the corresponding error is exponentially small in T. An attempt to expand the integral (6) in terms of the series of the exponential function $e^{-t}=\sum_{k=0}^{\infty}\frac{(-1)^k t^k}{k!}$ yields a divergent series; however, this is not the case for the truncated transform (7). We find the following series expansion:

$$\tilde{S}_{su}(x,T)=\frac{1}{2}\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}x^k e^{\frac{k^2\sigma^2}{2}}erfc(\frac{k\sigma}{\sqrt{2}}+\frac{\ln(x/T)}{\sigma\sqrt{2}}) \qquad (8)$$

Similar the PDF of the Suzuki random variable may be found from (6) by differentiation:

$$s_{su}(x)=-\tilde{S}_{su}'(x)=\int_{t=0}^{\infty}e^{-xt}ts_{\ln}(t)dt=\frac{1}{\sqrt{2\pi}\sigma x}\int_{t=0}^{\infty}e^{-t-\frac{(\ln(t/x))^2}{2\sigma^2}}dt \qquad (9)$$

and for the PDF we now we take the corresponding truncated integral to be:

$$s_{su}(x,T)=\frac{1}{\sqrt{2\pi}\sigma x}\int_{t=0}^{T}e^{-t-\frac{(\ln(t/x))^2}{2\sigma^2}}dt \qquad (10)$$

In this case we find $0\le s_{su}(x)-s_{su}(x,T)\le e^{-T+\frac{\sigma^2}{2}}$ as a bound of the truncation error.

By expanding the integral (10) in terms of the exponential function as above, we now obtain a similar (convergent) series:

$$s_{su}(x,T)=\frac{1}{2}\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}x^k e^{\frac{(k+1)^2\sigma^2}{2}}erfc(\frac{(k+1)\sigma}{\sqrt{2}}+\frac{\ln(x/T)}{\sigma\sqrt{2}}) \qquad (11)$$

D. Distribution of the obtainable bitrate a user located at a given distance from the sender antenna

Below we express the distribution of the obtainable bitrate according to the distribution of the stochastic part of the SINR; namely S_t. From (1) we get the bit-rate $B_t(r)$ for a channel occupying a bandwidth f located at distance r as:

$$B_t(r)=f\,c_j \text{ when } S_t\in\left[h(r,\lambda)g_j, h(r,\lambda)g_{j+1}\right);\ j=0,1,...,15 \qquad (12)$$

Hence, the DF (Distribution Function) of the bandwidth for a user located at distance r; $B(y,r)=P(B_t(r)\le y)$ may be written:

$$B(y,r)=S(h(r,\lambda)g_{j+1}) \text{ for } y\in\left(f\,c_j, f\,c_{j+1}\right];\ j=0,1,...,15 \qquad (13)$$

where $S(x)$ is the DF of the stochastic variable fading component. Also we obtain the k'moment of the obtainable bitrate for a user located at a distance r from the antenna as the (finite) sum:

$$m_k(r)=f^k\sum_{j=1}^{15}\left(c_j^{\,k}-c_{j-1}^{\,k}\right)\tilde{S}(h(r,\lambda)g_j) \qquad (14)$$

where $\tilde{S}(x)=1-S(x)$ is the CDF of the stochastic variable fading component.

E. Distribution of the obtainable bitrate for a user that is randomly placed in a circular cell with power-law attenuation

Since the bitrate/capacity for a user strongly will depend of the distance from the sender antenna, a better measure of the capacity will be to find the distribution of bitrate for a user that is randomly located in the cell. This is done by averaging over the cell area and the distribution of the corresponding averaging bitrate B_t is given as $B(y)=\frac{1}{A}\int_A B(y,r)dA(r)$ where A is the cell area. For circular cell shape with radius R, and power law attenuation on the form $h(r,\lambda)=h(\lambda)r^{\alpha}$ the corresponding integrals may be partly evaluated. By defining an α-factor averaging random variable S_α with DF $S_\alpha(x)=P(S_\alpha\le x)$ defined by

$$S_\alpha(x)=\frac{2}{\alpha}x^{-\frac{2}{\alpha}}\int_{t=0}^{x}t^{\frac{2}{\alpha}-1}S(t)dt=\frac{2}{\alpha}\int_{t=0}^{1}t^{\frac{2}{\alpha}-1}S(tx)dt \qquad (15)$$

and with PDF

$$s_\alpha(x)=\frac{2}{\alpha}x^{-\frac{2}{\alpha}-1}\int_{t=0}^{x}t^{\frac{2}{\alpha}}s(t)dt=\frac{2}{\alpha}\int_{t=0}^{1}t^{\frac{2}{\alpha}}s(tx)dt \qquad (16)$$

the bitrate distribution will have exact the same form as (13), and with moments given by (14) by changing $r\to R$ and $S(x)\to S_\alpha(x)$ (and $s(x)\to s_\alpha(x)$) in the formulas.

1) Distribution of the stochastic variable S_α for Log-normal and Suzuki distribution

Based on the definition we may derive the CDF and PDF of stochastic variable S_α for the Log-normal and Suzuki distributed fading models. For the Log-normal distribution we have $\tilde{S}_{\ln\alpha}(x)=\frac{1}{\alpha\, x^{2/\alpha}}\int_{t=0}^{x}t^{2/\alpha-1}erfc(\frac{\ln t}{\sigma\sqrt{2}})dt$. By changing variable according to $y=\ln t$ in the integral we find:

$$\tilde{S}_{\ln\alpha}(x)=\frac{1}{2}\left(erfc(\frac{\ln x}{\sigma\sqrt{2}})+x^{-2/\alpha}e^{2\sigma^2/\alpha^2}erfc(\frac{2\sigma^2-\alpha\ln x}{\alpha\sigma\sqrt{2}})\right) \qquad (17)$$

and further the PDF is found by differentiation:

$$s_{\ln\alpha}(x)=\frac{1}{\alpha}x^{-(2/\alpha+1)}e^{2\sigma^2/\alpha^2}erfc(\frac{2\sigma^2-\alpha\ln x}{\alpha\sigma\sqrt{2}}) \qquad (18)$$

For the Suzuki distribution we have the CDF given by the integral $\tilde{S}_{su}(x)=x\int_{t=0}^{\infty}t^{-2}e^{-t}s_{\ln}(x/t)dt$ and therefore we have:

$$\tilde{S}_{su\,\alpha}(x)=\frac{2}{\alpha}\int_{t=0}^{1}t^{\frac{2}{\alpha}-1}\tilde{S}_{su}(xt)dt=x\int_{t=0}^{\infty}t^{-2}e^{-t}s_{\ln\alpha}(x/t)dt \qquad (19)$$

where $s_{\ln\alpha}(x)$ is given by (18) above for the Lognormal distribution. As for the Suzuki distribution approximation to any accuracy is possible to obtain by truncating the integral above:

$$\tilde{S}_{su\,\alpha}(x,T)=x\int_{t=0}^{T}t^{-2}e^{-t}s_{\ln\alpha}(x/t)dt \qquad (20)$$

and also for this case we find that the truncation error is exponentially small in T. By expanding $e^{-t}=\sum_{k=0}^{\infty}\frac{(-1)^k t^k}{k!}$ and integrating term by term we find:

$$\tilde{S}_{su\,\alpha}(x,T)=\sum_{k=0}^{\infty}\frac{(-1)^k x^k}{(2+k\alpha)k!}e^{\frac{k^2\sigma^2}{2}}erfc(\frac{k\sigma}{\sqrt{2}}+\frac{\ln(x/T)}{\sigma\sqrt{2}})+ \frac{e^{2\sigma^2/\alpha^2}}{\alpha}\gamma(\frac{2}{\alpha},T)x^{-2/\alpha}erfc(\frac{2\sigma^2-\alpha\ln(x/T)}{\alpha\sigma\sqrt{2}}) \qquad (21)$$

where $\gamma(a,x)=\int_{t=0}^{x}t^{a-1}e^{-t}dt$ is the incomplete gamma function. (Observe the similarity with the corresponding expansion for $\tilde{S}_{su}(x)$ by (8).)

The corresponding integral for the PDF is given by:

$$s_{su\,\alpha}(x)=\int_{t=0}^{\infty}t^{-1}e^{-t}s_{\ln\alpha}(x/t)dt \qquad (22)$$

and we take the truncated approximation of the PDF as the integral:

$$s_{su\,\alpha}(x,T)=\int_{t=0}^{T}t^{-1}e^{-t}s_{\ln\alpha}(x/t)dt \qquad (23)$$

and we find the following error bound:

$0\le s_{su\,\alpha}(x)-s_{su\,\alpha}(x,T)\le e^{-T+\frac{\sigma^2}{2}}$. By the similar approach as for the CDF we also find the following series expansion of the truncated PDF:

$$s_{su\,\alpha}(x,T)=\sum_{k=0}^{\infty}\frac{(-1)^k x^k}{(2+(k+1)\alpha)k!}e^{\frac{(k+1)^2\sigma^2}{2}}erfc(\frac{(k+1)\sigma}{\sqrt{2}}+\frac{\ln(x/T)}{\sigma\sqrt{2}})+ \frac{e^{2\sigma^2/\alpha^2}}{\alpha}\gamma(\frac{2}{\alpha}+1,T)x^{-(1+2/\alpha)}erfc(\frac{2\sigma^2-\alpha\ln(x/T)}{\alpha\sigma\sqrt{2}}) \qquad (24)$$

III. Estimation of cell capacity

In the following we assume that the cell is loaded by two traffic types:

- High priority GBR traffic sources that each requires to have a fixed data-rate and
- low priority (greedy) data sources that always consumes the leftover capacity not used by the GBR traffic.

This is actual a very realistic traffic scenario for future LTE networks where we actual will have a mixture of both real time traffic like VoIP and elastic data traffic. Below, we first estimate the Resource Block (RB) usage of the high priority GBR traffic, and then we may subtract the corresponding RBs to find the actual numbers of RBs available for the (greedy) data traffic sources. Then finally we estimate the cell throughput/capacity as the sum of the bitrates offered to the GBR and (greedy) data sources.

A. Estimation of the capacity usage for GBR sources in LTE

The reservation strategy considered simply reserve recourses on a per TTI bases and allocate RBs so that the aggregate rate equals the required GBR rate, i.e. using Non-Persistent scheduling.

1) Capacity usage for a single GBR source

We first consider the case where we know the location of the GBR user in the cell, i.e. is located at a distance r from the antenna. We take B as the bitrate obtainable for a single RB and consider a GBR source that requires a fixed bit-rate of b^{CBR}. We assumes that this is achieved by offering n RBs for every k-TTI interval. A simple way of reserving resources to GBR connections is to allocate RBs so that $\frac{n}{k}B$ will be close to the required rate b^{CBR} over a given period. We take $N_{CBR}=\frac{n}{k}$ to be the number of RBs granted for a GBR connection in a TTI as (the stochastic variable):

$$N_{CBR} = \begin{cases} \frac{\alpha b^{CBR}}{B} & if \quad CQI > 0 \\ 0 & if \quad CQI = 0 \end{cases} \quad (25)$$

where we have introduced a scaling factor α so that on the long run we obtain the desired GBR-rate b^{CBR}. By choosing $\alpha = p_{CQI}^{-1}$ where $p_{CQI} = P(CQI > 0) = \tilde{S}(h(r,\lambda)g_1)$ then $E[N_{CBR}B] = b^{CBR}$ and hence we also have:

$$E[N_{CBR}|CQI > 0] = \frac{b^{CBR}}{p_{CQI}} E[B^{-1}|CQI > 0] \quad (26)$$

The mean numbers of RBs is therefore:

$$\beta = \beta(r, b^{CBR}) = b^{CBR} m_{-1}^{CQI}(r) \quad (27)$$

where the conditional moments $m_k^{CQI}(r) = E[B^k|CQI > 0]$ is found as

$$m_k^{CQI}(r) = \frac{f^k}{\tilde{S}(h(r,\lambda)g_1)}\left(c_1^k \tilde{S}(h(r,\lambda)g_1) + \sum_{j=2}^{15}\left(c_j^k - c_{j-1}^k\right)\tilde{S}(h(r,\lambda)g_j)\right) \quad (28)$$

Note that by conditioning on having $CQI > 0$ we exclude the users that are unable to communicate due to bad radio conditions, and avoid the problems due to division of zero in the calculation of the mean of $1/B$. For circular cells and power law attenuation we obtain the corresponding result as above by changing $r \rightarrow R$ and $S(x) \rightarrow S_\alpha(x)$.

2) Estimation of RBs usage for several GBR sources

In the following we first estimate the RB usage for a fixed number of M GBR sources located at distances r_j from the antenna with bit-rate requirements b_j^{CBR}; $j = 1,...,M$. The total usage of RBs β^{CBR} will be the sum the individual contribution from each source as given by (27):

$$\beta^{CBR} = \sum_{j=1}^{M} \beta(r_j, b_j^{CBR}) \quad (29)$$

For the case with random location the expression gets even simpler:

$$\beta^{CBR} = \beta(R, \sum_{j=1}^{M} b_j^{CBR}) \quad (30)$$

i.e. we may add the GBR rates from all the sources in the cell. The corresponding throughput for the GBR sources is taken as the sum of the individual rates i.e.

$$b^{CBR} = \sum_{j=1}^{M} b_j^{CBR} \quad (31)$$

B. Estimation of the capacity usage for a fixed number of greedy sources

In the following we shall estimate the capacity usage for a fixed number of greedy sources by assuming:

- There are totally K active (greedy) users that are placed randomly in the cell and always have traffic to send, i.e. we consider the cell in saturated condition.
- There are totally N available RBs and the scheduled user is granted all of them in a TTI interval.

1) Scheduling of based on metrics

First we consider the case with K users that are located in the cell with distances (from the sender antenna) given by a vector $\mathbf{r} = (r_1,....,r_K)$ and we assume that the user scheduled in a TTI is based on:

$$i_{schedul} = \arg\max_{i=1,...,K}\{M_i\} \quad (32)$$

where $M_i = M_i(\mathbf{r})$ is the scheduling metric which also may depend on the location of all users (through the location vector $\mathbf{r} = (r_1,....,r_K)$). Hence, for the scheduler to choose a user i, the metric M_i must be larger than all the other metrics (for the other users), i.e. we must have $M_i > U_i$ where

$$U_i = \max_{\substack{k=1,...,K \\ k \neq i}} M_k \quad (33)$$

Since we assume that a user is granted all the RBs when scheduled, this gives the cell throughput when user i is scheduled (located at distance r_i) to be $NB(r_i)$, where $B(r_i)$ is the corresponding obtainable bit-rate per RB. Hence, cell bit-rate distribution may then be written as:

$$B_g(y,\mathbf{r}) = \sum_{i=1}^{K} B_i(y,\mathbf{r}) \text{ where} \quad (34)$$

$$B_i(y,\mathbf{r}) = P(NB(r_i) \leq y, M_i(\mathbf{r}) > U_i(\mathbf{r})) \quad (35)$$

is bitrate distribution when user i is scheduled. Unfortunately, in the general case, explicit expression of the probabilities $B_i(y,\mathbf{r})$ is difficult to obtain, mainly due to the involvement of the scheduling metrics. However, for some cases of particular interest closed form analytical expression is possible to obtain. For many scheduling algorithms the scheduling metrics are only function of the SINR for that particular user (and does not depend of the SINR for the other users) and for this case extensive simplification is possible to obtain. In the following we therefore assume that the metrics M_i only are functions of their own $SINR_i$ and the location r_i of that particular user, i.e. we have $M_i = M(S_i, r_i)$, where we (for simplicity) also assume that $M(x, r_i)$ is an increasing function of x with an unlikely defined inverse function $M^{-1}(x, r_i)$. The distribution functions for M_i and $U_i = \max_{\substack{k=1,...,K \\ k \neq i}} M_k$ are then

$$M_i(x, r_i) = P(M_i \leq x) = S(M^{-1}(x, r_i)) \text{ and} \quad (36)$$

$$U_i(x,\mathbf{r}) = P(U_i \leq x) = \prod_{k=1,k\neq i}^{K} S(M^{-1}(x, r_k)) \quad (37)$$

If we now condition on the value of $S_i = x$ in (35), we find the distribution of the cell capacity when user i is scheduled as:

$$B_i(y,\mathbf{r}) = \int_{x=0}^{\infty} P(B(r_i) \leq \frac{y}{N}|S_i = x) U_i(M(x, r_i), \mathbf{r}) s(x) dx \quad (38)$$

By using (12) as the obtainable bit-rate per RB we find:

$$B_i(y,\mathbf{r}) = \int_{x=0}^{h(r_i,\lambda)g_{j+1}} F_i(x,\mathbf{r})s(x)dx \; if \; y/N \in (fc_j, fc_{j+1}]; \; j = 0,1,...,15 \quad (39)$$

where we now have defined the multiuser "scheduling" function $F_i(x,\mathbf{r})$ by:

$$F_i(x,\mathbf{r}) = U_i(M(x,r_i),\mathbf{r}) = \prod_{k=1,k\neq i}^{K} S(M^{-1}(M(x,r_i),r_k)) \quad (40)$$

Finally, by assuming that all users are randomly located throughout the cell the corresponding bit-rate distribution is found by performing a K-dimensional averaging over all possible distance vectors $\mathbf{r}$ over the cell; $B_g(y) = \frac{1}{A^K}\int_A \ldots \int_A r_1 \ldots r_K B_{cell}(y,\mathbf{r})dA_1 \cdots dA_K$, where A here is the cell area. Due to the special form of the function $F_i(x,\mathbf{r}) = \prod_{k=1,k\neq i}^{K} S(M^{-1}(M(x,r_i),r_k))$ the "cell averaging" over the $K-1$ dimension variables $r_1,..,r_{i-1},r_{i+1},...,r_K$ (not including the variable r_i) yields the product $\left[\hat{S}(M(x,r_i)\right]^{K-1}$ where

$$\hat{S}(y) = \frac{1}{A}\int_A uS(M^{-1}(y,u))dA(u) \quad (41)$$

Hence, for the case when user i is located at distance r and all the $K-1$ other users located at random, then we find $B_i(y,r) = B(y,r)$ is independent of i and:

$$B(y,r) = \int_{x=0}^{h(r_i,\lambda)g_{j+1}} \left[\hat{S}(M(x,r))\right]^{K-1} s(x)dx \; if \; y/N \in (fc_j, fc_{j+1}]; \quad (42)$$

$j = 0,1,...,15$,

and moreover $p(r) = \lim_{y\to\infty} B_i(y,r) = \int_{x=0}^{\infty}\left[\hat{S}(M(x,r_i))\right]^{K-1} s(x)dx$ is the probability that a particular user is scheduled.

For circular cell size the cell bit-rate distribution is therefore given as:

$$B_g(y) = \frac{2}{R^2}\int_{r=0}^{R} r \int_{x=0}^{h(r,\lambda)g_{j+1}} K\left[\hat{S}(M(x,r))\right]^{K-1} s(x)dxdr \; if \; y/N \in (fc_j, fc_{j+1}]; \quad (43)$$

$j = 0,1,...,15$

where we now have

$$\hat{S}(y) = \frac{2}{R^2}\int_{r=0}^{R} uS(M^{-1}(y,u))du \quad (44)$$

The moments of the cell capacity (when the users are located according to the vector $\mathbf{r} = (r_1,....,r_K)$ may be written as:

$$E[B_g(\mathbf{r})^k] = f^k N^k \sum_{i=1}^{K}\sum_{j=1}^{15} c_j{}^k \int_{x=h(r_i,\lambda)g_j}^{h(r_i,\lambda)g_{j+1}} F_i(x,\mathbf{r})s(x)dx \quad (45)$$

The corresponding moments for cell capacity when the users are randomly located in a circular cell is given by:

$$E[B_g{}^k] = \frac{2f^kN^k}{R^2}\sum_{j=1}^{15} c_j{}^k \int_{r=0}^{R} r\left\{\int_{x=h(r,\lambda)g_j}^{h(r,\lambda)g_{j+1}} K\left[\hat{S}(M(x,r))\right]^{K-1} s(x)dx\right\}dr \quad (46)$$

2) Examples

Below we consider and compare three of the most commonly known scheduling algorithms, namely Round Robin (RR), Proportional Fair (PF) and Max SINR by applying the cell capacity models described above.

a) Round Robin

For the Round Robin algorithm each user is given the same amount of bandwidth and hence this case corresponds to taking $K = 1$, i.e. the results in section II may be applied by to find the cell capacity with $f \to Nf$ and $S(x) \to S_{su}(x)$ and also $S_\alpha(x) \to S_{su\,\alpha}(x)$.

b) Proportional Fair (in SINR)

Normally, the shadowing is varying over a much longer time scale than the TTI intervals, and hence we may assume that the slow fading is constant during the updating of the scheduling metric M_i and therefore should only account for the rapid fading component. This means that the shadowing effect may be taken as constant that may be included in the non varying part of the SINR over several TTI intervals. Hence, we take SINR as $S_t/h(r,\lambda)$ where $S_t = zX_e$ conditioned that the shadowing $X_{\ln} = z$. By assuming $X_{\ln} = z$ is constant over the short TTI intervals, the scheduling metrics will be $M_i = \frac{zX_e/h(r_i,\lambda)}{zE[X_e]/h(r_i,\lambda)} = \frac{X_e}{E[X_e]}$. In the final result we then "integrate over the Log-normal slow fading component". We find that the probability of being scheduled is $p(r) = \frac{1}{K}$ and that the conditional bandwidth distribution for a user at located at distance r (and the $K-1$ users random located) is given by the results in section II.D with $f \to Nf$ and $S(x) \to S_K(x)$ with:

$$S_K(x) = \int_{t=0}^{\infty} S_e(\frac{x}{t})^K s_{\ln}(t)dt \text{ and } s_K(x) = \int_{t=0}^{\infty} \frac{K}{t} S_e(\frac{x}{t})^{K-1} s_e(\frac{x}{t}) s_{\ln}(t)dt \quad (47)$$

where $S_e(x) = 1 - e^{-x}$ and $s_e(x) = e^{-x}$.

Further the distribution of the cell capacity is given by the results in section II.E with $f \to Nf$ and further the corresponding α-averaging is given by the integrals:

$$\tilde{S}_{K\alpha}(x) = \int_{t=0}^{\infty} KS_e(t)^{K-1} s_e(t)\tilde{S}_{\ln\alpha}(x/t)dt \text{ and} \quad (48)$$

$$s_{K\alpha}(x) = \int_{t=0}^{\infty} KS_e(t)^{K-1} s_e(t)t^{-1} s_{\ln\alpha}(x/t)dt \quad (49)$$

c) Max SINR algorithm.

For this algorithm the scheduling metric is $M_i = S_i/h(r_i,\lambda)$. By assuming circular cell size and radio signal attenuation on the form $h(r,\lambda) = h(\lambda)r^\alpha$ gives:

$$\hat{S}(M(x,r)) = \frac{2}{R^2}\int_{r=0}^{R} uS(x(u/r)^\alpha)du = S_\alpha(x(R/r)^\alpha) \quad (50)$$

We find that the probability of being scheduled

$$p(r)=\int_{x=0}^{\infty}\left[S_{\alpha}(x(R/r)^{\alpha})\right]^{K-1}s(x)dx \tag{51}$$

and that the conditional bandwidth distribution for a user located at distance r (and the $K-1$ other users random located) is given by the results in section II.D with $f \rightarrow Nf$ and $S(x)\rightarrow S_c(x,r)$ with:

$$S_c(x,r)=\frac{1}{p(r)}\int_{y=0}^{x}\left[S_{\alpha}(y(R/r)^{\alpha})\right]^{K-1}s(y)dx \tag{52}$$

It turns out that extensive simplifications occur for the case where all the users are randomly located in the cell and we find that the distribution of the cell capacity is given by the results in section II.E with $f \rightarrow Nf$ and further the α -averaging is given by taking $S_{\alpha}(x)\rightarrow S_{\alpha}(x)^K$ i.e. is simply the K 'th power of the α -averaging of $S(x)$.

C. Combining real-time and non real time traffic over LTE

We are now in the position to combine the analysis in sections III.A and III.B to obtain complete description of the resource usage in a LTE cell. The combined modeling is based on the following assumptions: There are M GBR sources applying the allocation described in section III.A and K active (greedy) data sources which always have traffic to send, i.e. we consider the cell in saturated conditions. Further the number of available RBs is taken to be N.

Since the GBR sources have priority over the data sources, they will always get the number of RBs they need and hence the leftover RBs will be available for the Non-GBR data sources. By conditioning on the RB usage of the GBR sources we may apply all the results derived in section III.B with available RBs taken to be the leftover RBs not used by the GBR sources. Then we may find the average usage of RBs for the GBR traffic as done in section III.A.

We consider first the case where the location of the sources is given; where the GBR sources are located at distances s_j from the antenna with bit-rate requirements b_j^{CBR}, $j=1,...,M$, and the greedy data sources are located at distance r_i, $i=1,...,K$.

With these assumptions the mean cell throughput is given as:

$$B_{cell}=f(N-\sum_{j=1}^{M}\beta(s_j,b_j^{CBR}))\sum_{i=1}^{K}\sum_{j=1}^{15}c_j\int_{x=h(r_i,\lambda)g_j}^{h(r_i,\lambda)g_{j+1}}F_i(x,\mathbf{r})s(x)dx+\sum_{j=1}^{M}b_j^{CBR} \tag{53}$$

where $\beta(r,b^{CBR})$ is given by (27) and further $F_i(x,\mathbf{r})$ is defined by (40). For circular cells and power law attenuation on the form $h(r,\lambda)=h(\lambda)r^{\alpha}$ and randomly placed sources the corresponding cell throughput is found to:

$$B_{cell}=f(N-\beta(R,\sum_{j=1}^{M}b_j^{CBR}))\frac{2}{R^2}\sum_{j=1}^{15}c_j\int_{r=0}^{R}r\left\{\int_{x=h(r,\lambda)g_j}^{h(r,\lambda)g_{j+1}}K\left[\widehat{S}(M(x,r))\right]^{K-1}s(x)dx\right\}dr+\sum_{j=1}^{M}b_j^{CBR} \tag{54}$$

where $\beta=\beta(r,b^{CBR})$ is given by (27) and further $\widehat{S}(M(x,r))$ is defined by (44). Observe that the GBR traffic only will affect the cell throughput by the sum $\sum_{j=1}^{M}b_j^{CBR}$ of the rates and not the actual number of GBR sources.

IV. Discussion of numerical examples

In the following we give some numerical example of downlink performance of LTE based on the models described in sections II and III.

TABLE 2 INPUT PARAMETERS FOR THE NUMERICAL CALCULATIONS

Parameters	Numerical values
Bandwidth per Resource Block	180 kHz=12x 15 kHz
Total Numbers of Resource Blocks	100 RBs for 2Ghz
Distance-dependent path loss. (The actual model is found in [6].)	L=C +37.6log10(r), r in kilometers and C=128.1 dB for 20MHz
Lognormal Shadowing with standard deviation	8 dB (in most of the cases)
Rayleigh fast fading	
Noise power at the receiver	-101 dBm
Total send power	46.0 dBm=(40W)
Radio signaling overhead	3/14

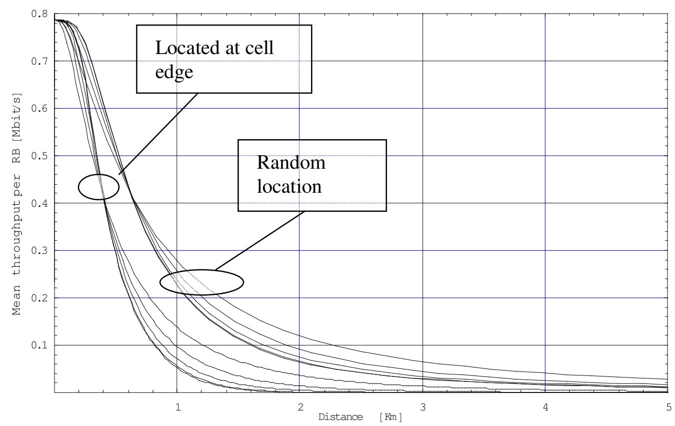

Figure 1: Mean throughput right and std. left per RB for a user random located, and fixed located as function of cell radius with 2 GHz sending frequency and Suzuki distributed fading with std. of fading σ=0dB, 2dB, 5dB, 8dB, 12dB from below.

In Figure 1 we have depicted the mean bitrate per RB as function of the cell radius. The maximum bitrate is just below 0.8 Mbit/s for excellent radio conditions. The mean bitrate per RB have decreased to 0.1 Mbit/s for cell sizes of approximately 2 km and shadowing std. equal 8 dB and when users are random located. The corresponding bit-rate for users at the cell edge is approximately 0.04 Mbit/s.

As seen from Figure 2 the Max-SINR algorithm will over perform the PF algorithm when it comes to cell throughput. But if we consider fairness among users the picture is complete different. When considering the performance of users located at cell edge the Max-SINR algorithm actually performs very badly. While PF give equal probability of transmitting in a TTI for all active users, the Max-SINR strongly discriminate the user close to cell edge.

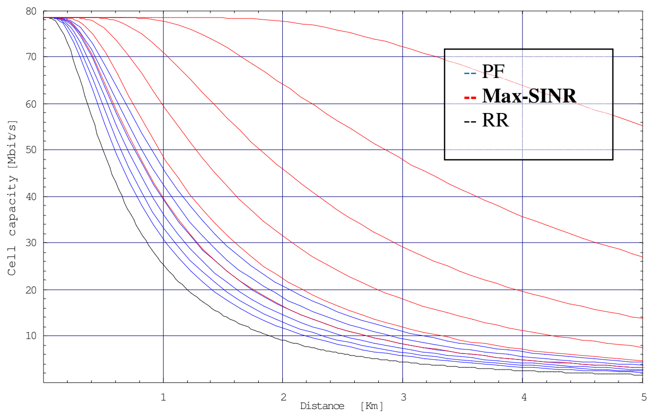

Figure 2: Cell capacity as function of cell radius for Max-SINR (red), PF (blue) and RR (black) scheduling, 2GMHz frequency with 100 RB and with Suzuki distributed fading with std. σ=8 dB. The number of users is 1, 2, 3, 5, 10, 25, 100 from below.

As seen from Table 3 below, for the case with 10 active users in a cell, the PF fairly give each user 10% chance of accessing radio resource while the Max-SINR only give 1.2% chance of accessing the radio resources if a user is located at cell edge.

TABLE 3: PROBABILITY THAT A USER IS SCHEDULED AS FUNCTION OF NUMBERS OF USERS AND LOCATION FOR PF AND MAX-SINR SCHEDULING ALGORITHMS, AND SUZUKI DISTRIBUTED FADING WITH STD. OF 8DB.

Number of users	PF	MAX-SINR			
		r/R=1	r/R=0.5	r/R=0.25	r/R=0.1
2	0.50	0.308708	0.594756	0.82579	0.96119
3	0.33	0.147869	0.414839	0.71126	0.92784
5	0.20	0.055113	0.245871	0.56102	0.87130
10	0.10	0.012690	0.104912	0.36531	0.76418
25	0.04	0.001356	0.025222	0.16326	0.56989
100	0.01	0.000019	0.001293	0.02453	0.24325

In Figure 3 we consider the cases where 10 greedy users are scheduled by the PF algorithm together with a GBR user with guaranteed rate of 3, 1, 0.3 or 0.1 Mbit/s. We consider the cases where either the GBR user is located at cell edge or have random location throughout the cell. We observe that thin GBR connections do not have big impact on the cell throughput, and it seems that GBR bearers up to 1 Mbit/s should be manageable without influencing the cell performance very much. Higher GBR rates, however, (i.e. 3 Mbit/s) will reduce the cell throughput by a quite large factor especially if the user is located near the cell edge.

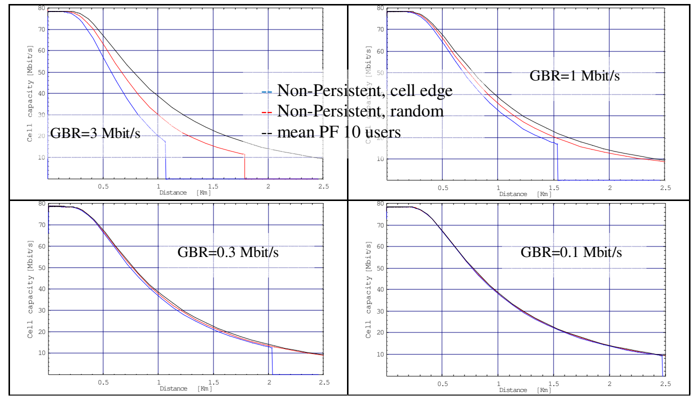

Figure 3: Mean cell throughput for PF, 10 users and a GBR user of 3.0, 1.0, 0.3, 0.1 Mbit/s using non-persistent scheduling, for 2 GHz and 100 RB and Suzuki distributed fading with std. σ=8dB. Red curves corresponds to random location and blue for user located at cell edge.

V. CONCLUSIONS

By extensive analytical modeling where both fading and attenuation due distance is included we obtain performance models for:

- Spectrum efficiency through the bitrate distribution per RB for users that are either randomly or located at a particular distance in a cell.
- Cell throughput/capacity and fairness by taking the scheduling into account.
- Cell throughput/capacity for a mix of GBR and Non-GBR (greedy) users.

The usage of GBR with high rates may cause problems in LTE due to the high demand for radio resources if users have low SINR i.e. at cell edge. For GBR allocation the allowed guaranteed rate should be limited. It seems that a limit close to 1 Mbit/s will be a good choice.

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