On Modeling of Fluctuations in Quasi-Static Approach Describing the Temporal Evolution of Retry Traffic

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***Abstract*—We previously introduced a traffic model that describes the behavior of the retry traffic created by users who are impatient when waiting for a service to be provided. The behavior can be described in a simple form if it is assumed that the system offers infinitely fast processing (i.e. ideal). Moreover, we proposed the quasi-static approach that replicates the temporal evolution of traffic in finite speed (i.e. actual) systems. In the quasi-static approach, the difference between the behavior of the ideal system and that of the actual system is treated as stochastic fluctuation. However, work presented to date not verified that the quasi-static approach can express the traffic model. This paper calculates the temporal evolution of traffic in the M/M/1 based model with retry traffic by traditional Monte-Carlo simulations and the quasi-static approach. The results show that quasi-static approach is as good as the traditional approach in modeling the traffic.**

I. Introduction

Significant problems with the Internet include node failure due to congestion or overload. One of the key factors behind overload is the generation of retry traffic. Therefore, to optimize resource allocation and construct stable systems, an evaluation method that can accurately model retry traffic is important.

We consider user impatience which is one of the cause of retry traffic. Users who can not endure their waiting time before starting the service might generate duplicate service requests. In this study, we focus on one of the simplest model as an example: M/M/1 with retry traffic. Our model is M/M/1 that is added retry traffic, and the rate of the retry traffic is proportional to the queue length (see Fig. 1).

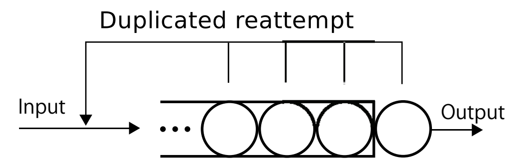

Fig. 1. Model with the retry traffic that is proportional to the M/M/1 queue.

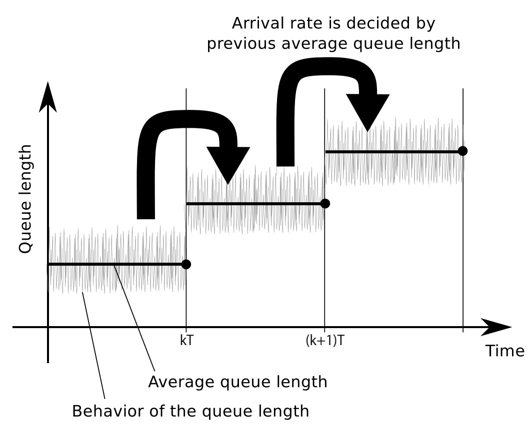

Fig. 2. Relationship between a change in arrival rate and the queue length

[1] modeled the behavior of retry traffic due to impatience; the result is called the *quasi-static retry traffic model*. [1] separates a timescale of the transitions of retry traffic and queue length because the change of the queue is very faster compared with the user responses. Therefore, the change of the traffic rate is proportional to not the queue length at the present but average queue length on past T period (see Fig. 2). If the change of queue is infinitely fast, the average queue length is fixed and the change of the retry traffic is obtained easily.

Since an actual system does not work at infinite high speed, [1] proposed the *quasi-static approach* to evaluate the behavior of traffic on systems that offered actual finite speeds. In the quasi-static approach, the difference between the behavior of the infinitely high-speed system and that of the finite speed system is treated as stochastic fluctuation. Compared to the traditional approaches, the quasi-static approach might have a superiority when we estimate the small probability of traffic divergence on high-speed systems.

In the quasi-static approach, we describe the traffic of the target system as the following Langevin equation:

$$\frac{\mathrm{d}}{\mathrm{d}t}X(t) = F(X(t)) + \sqrt{D(X(t))}\,\xi(t)\,, \qquad (1)$$

where $X(t)$ and $\xi(t)$ denote the number of arrivals in past T period and the white Gaussian noise, respectively. The first and second terms of (1) represent the behavior of the infinitely

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high-speed system and the stochastic fluctuation, respectively. According to [1], in the case of the model in Fig. 1, $F(X(t))$ and $D(X(t))$ are given as follows:

$$F(X(t)) = \lambda_0 - \frac{X(t)}{T} + \varepsilon \frac{X(t)/(\mu T)}{1 - X(t)/(\mu T)}, \qquad (2)$$

$$D(X(t)) = \lambda_0 + \varepsilon \frac{X(t)/(\mu T)}{1 - X(t)/(\mu T)}, \qquad (3)$$

where λ_0 and μ denote the arrival rate excluding the retry traffic and the service rate of the system, respectively, and ε denotes the intensity of retry traffic.

Since [1] did not confirm the validity of quasi-static approach enough, we compute the temporal evolution of traffic of the above-mentioned model, and compare the result with that of Monte-Carlo simulations. The result show the validity of our approach though we must modify it.

II. Comparing the Quasi-static Approach to the Monte-Carlo Simulation

We verify that quasi-static approach, which adds random fluctuations to the behavior of infinitely high-speed system, can appropriately describe the behavior of actual finite speed systems.

It is well known that the Langevin equation (1) is equivalent to the Fokker-Planck equation as shown by

$$\frac{\partial}{\partial t} p(x,t) = -\frac{\partial}{\partial x} F(x) p(x,t) + \frac{1}{2} \frac{\partial^2}{\partial x^2} D(x) p(x,t), \qquad (4)$$

where $p(x,t)$ denotes the probability density function (pdf) of $X(t)$. By using (4), we can simulate the transition of the pdf of the volume of traffic, and can assess the probability of its divergence etc.

To confirm the validity of quasi-static approach, we compute the distribution of traffic volume at time t by Monte-Carlo simulation, and compare it with the result of the quasi-static approach. Results are shown in Fig. 3. The line of unmodified Fokker-Planck represents the result of Fokker-Planck equation by substituting (2) and (3) for (4). The parameters of the simulation are $\lambda_0 = 300$, $\mu = 1000$, $\varepsilon = 200$, $T = 1$ s, and $t = 50$ s. According to the figure, the distributions that are computed by the Monte-Carlo and unmodified Fokker-Planck are not corresponding. Therefore, we must reconsider (2) and (3).

Left-hand side of (1) expresses the change of $X(t)$, which is the number of arrivals on past T period, and $F(X(t))$ and $D(X(t))$ correspond to the mean and variance of it. The change of $X(t)$ is composed of the increment $U(t, \mathrm{d}t)$ and the decrement $-U(t-T, \mathrm{d}t)$, where $U(t,s)$ is the actual number of arrivals in the time interval $(t, t+s]$ (see Fig. 4). The mean and variance of random variable $U(t, \mathrm{d}t)$ are both $\lambda_0 \mathrm{d}t$ because the future arrivals follow a Poisson arrival. Moreover, the conditional distribution of $U(t-T, \mathrm{d}t)$, given that $U(t-T, T) = X(t)$, obeys a Binomial distribution $B(X(t), \mathrm{d}t/T)$ [2]. Therefore, the mean of $U(t-T, \mathrm{d}t)$ is $(X(t)/T)\mathrm{d}t$, and the variance of $U(t-T, \mathrm{d}t)$ is $(X(t)/T)\mathrm{d}t - (X(t)/T^2)(\mathrm{d}t)^2 \simeq (X(t)/T)\mathrm{d}t$. As a result, we can modify $D(X(t))$ as follows:

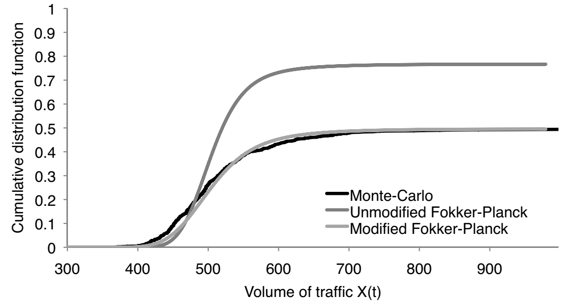

Fig. 3. Comparison of the cumulative distribution functions computed by Monte-Carlo simulation and Quasi-static approach.

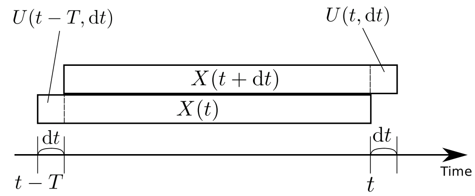

Fig. 4. Transition of $X(t)$ that expresses actual input traffic in past T period.

$$D(X(t)) = \lambda_0 + \frac{X(t)}{T} + \varepsilon \frac{X(t)/(\mu T)}{1 - X(t)/(\mu T)}. \qquad (5)$$

Note that $F(X(t))$ does not need modification.

In Fig. 3, we show the result of Fokker-Planck equation that is modified by (5). According to the figure, we can confirm that the quasi-static approach yields results similar to those of the Monte-Carlo simulation.

III. Conclusion

In this study, we verified the validity of quasi-static approach that describes the behavior of input traffic including retry traffic. We used the quasi-static approach to compute the temporal evolution of input traffic on a M/M/1 based system with retry traffic, and compared the result with that of a Monte-Carlo simulation. As a result, we confirmed that the quasi-static approach can appropriately evaluate a system with retry traffic.

Acknowledgment

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References

[1] M. Aida, C. Takano, M. Murata, and M. Imase, "A proposal of quasi-static approach for analyzing the stability of IP telephony systems," in *Proc. International Conference on Networking (ICN 2008)*, Cancun, Mexico, Apr. 2008, pp. 363–370.

[2] N. L. Johnson, A. W. Kemp, and S. Kotz, *Univariate discrete distributions*, 3rd ed. Wiley-Interscience, 2005, ch. 4, p. 166.