# Managing Spectrum into Abundance

Timothy K. Forde, Irene Macaluso & Linda E. Doyle
CTVR, University of Dublin, Trinity College, Ireland.

***Abstract*—The purpose of this paper is to propose a flexible spectrum auction that supports highly expressive bids. Essentially an expressive bid means that a bidder can ask for *what they really want* rather than select from a limited set of options. The paper presents a combinatorial auction that enables; variable, continuous, bidder defined, bandwidth offerings; variable, bidder defined, allocations of paired (FDD) and unpaired (TDD) spectrum; FDD assignments subject to a minimal, not absolute duplex gaps; and, variable, bidder defined, asymmetric FDD assignments. Bidders can request spectrum in OR, AND and XOR combinations. This results in an auction process that effectively allows bidders play a role both in the *allocation* and *assignment* process. Results are presented which show this type of auction leads to highly efficient use of spectral resources. In addition it is in keeping with more dynamic approaches which recognize that we are in a world of rapidly advancing technologies.**

## I. INTRODUCTION

Auctions are an increasingly common practice around the world as a regulatory tool for providing access to spectrum. They have been used for more than three decades in the US, and in more recent decades in Europe. An auction is a market-driven mechanism for the assignment of spectrum rights to those who value those rights most and as such the incorporation of auctions into spectrum management regimes move spectrum management techniques beyond command and control approaches.

In this paper we argue that, to date, auction processes have tended to underpin traditional notions of spectrum management because of how they are designed. We contend that the auction mechanisms in use today are constructed in a manner that embodies the practices of *managing spectrum into scarcity*, practices that reflect the legacy of the command and control world. This has a significant bearing on how efficiently spectrum can be utilized. We suggest an alternative design approach that incorporates more expressive bidding options into the auction process and that is not restricted by traditional ways of thinking about spectrum assignment. We aim to show that even though the same amount of spectrum is constantly available, if managed properly the appearance of an *abundance of spectrum*, if not at least the lack of scarcity, could be realized. The ideas here build on the longer term vision for cellular operators laid out in [1].

The first contribution of the paper is to show how the parameters of current auction processes are inflexible and not in keeping with advances in the field. The second and main contribution of this paper is an auction design that enables; variable, continuous, bidder defined, bandwidth offerings; variable, bidder defined, allocations of paired Frequency Division Duplexing (FDD) and unpaired Time Division Duplexing (TDD) spectrum; FDD assignments subject to a minimal, not absolute duplex gaps; and, variable, bidder defined, asymmetric FDD assignments. We present a working design in the form of a combinatorial clock auction. This is a significant advancement on current combinatorial practices that auction predefined non-variable 'lots' of spectrum. We demonstrate the effectiveness of the approach with a range of examples. Furthermore, we propose a technique that addresses the future spectrum needs of LTE-Advanced systems, taking account of its intended ability to handle inter- and intra-band carrier aggregation. The rest of this paper is laid out as follows. Section II presents the key parameters of the auction process. Section III describes the proposed auction is detail; the objectives of the auction, the price discovery phase and the winner-determination problem (WDP) algorithms. Section IV details the results and the paper is concluded in Section V.

## II. THE PARAMETERS OF THE AUCTION PROCESS

When auctioning spectrum there are a plethora of auction types that can be used. Irrespective of the type of auction in use however, there are a number of concepts which tend to underpin all approaches.

Firstly, the regulator thinks in terms of FDD or TDD. This results in an *allocation* of spectrum as one type or the other. Further restrictions are put in place in the case of FDD as the spectrum on offer is designated as uplink or downlink spectrum. Secondly, a minimum 'bid size' or 'lot size' is always set, i.e. the spectrum on offer is channelized in some manner. For example, cellular bands are typically sold in multiples of 5MHz. Thirdly, the lots are usually available in certain configurations. FDD provides a good example here. For FDD systems an absolute duplex gap is specified, i.e. the uplink and downlink lots must be so many Hertz apart. The duplex gap is defined on the basis of technology capabilities - hence the kind of technology that will be in use will tend to influence the definition of the gap.

A number of observations can be made. The allocation of spectrum as FDD or TDD stems from a static world in which interference is best managed by rigidly controlling who can be neighbors. From a technology perspective it seems to have much less relevance for a world of self-organizing systems in which neighboring networks can communicate and collaborate. If spectrum trading (changes of use and ownership) is factored in, then this allocation of spectrum also means that trades are restricted to swapping like with like. From a business perspective it also makes increasingly less sense. For example an argument can be made that the balance between FDD and TDD spectrum allocated to LTE technologies is not appropriate given the increasing popularity of TDD systems. And the emergence of combined FDD/TDD basestations speaks to an openness to configure as needed.

The channelization of the spectrum for auction begs a further set of comments. Firstly there is a chicken and egg question as to whether bands were planned as such to facilitate standards dependent on the selected channelizations, *or* whether the standards were designed to fit within the existing band channelizations [1]. Either way, the resource, spectrum, is trapped in a channelization regime which may not reflect the true demands of it. More importantly this has many implications for the outcome of the auction. Bidders must bid for channel assignments which accommodate their demand. But may end up bidding for more spectrum than they need because of the size of the channels. So, if the spectrum is channelized into 5MHz blocks, then a bidder that truly wants 12MHz must actually bid for 15MHz. This also has implications for the revenue which can be generated from the auction. The fixed channelization has also tended to mean that symmetric assignments of uplink and downlink resources result.[2] This is despite the fact that downlink traffic is far greater than uplink traffic [3]. This is a very wasteful approach and it is in fact surprising that this practice has dominated to date.

The overly static mindset is also obvious in other ways. The designating of spectrum either as uplink or downlink can lead to problems if, as the CEPT [4] report shows, it turns out to be more beneficial to reverse the traditional pairing for interference reasons. Here a predefined designation causes problems. The static defining of the duplex gap is also not good in heterogeneous situations in which the duplex gap will vary.

With these arguments in mind we therefore suggest it is time to design an auction mechanism that is not restrained by traditional thinking. More specifically we believe that the auction mechanism should not assign the allocated spectrum. The auction should infact combine the processes of allocation and assignment to be a truly market driven mechanism. We therefore propose to enable the following features in the auction mechanism:

- Variable, or bidder defined, bandwidth offerings.
- Variable, or bidder defined, allocations of paired (FDD) and unpaired (TDD) spectrum.
- FDD assignments subject to a minimal, not absolute, duplex gap.
- Variable, or bidder defined, asymmetric FDD demands.

This kind of auction obviously suggests a combinatorial approach. Typically combinatorial auctions supports bids to ask for different combinations expressed as (A AND B) or (A OR C). Our auction process must go well beyond that and allow for many more combinations of AND/OR/XOR as well as the ability to ask for 'anything from the following range'. Our approach can be said to be more *expressive*. An expressive market is one that allows buyers and sellers to ask, in so far as it can be accommodated in the market mechanism, for *whatever they want*, specifying constraints and conditions that suit their requirements. In fact it has been shown that the more expressive a market is, the more Pareto efficient it is [5]. Indeed in the following sections we will show that out approach leads to a more efficient outcome.

## III. THE AUCTION PROCESS

For the purposes of the paper we have chosen a combinatorial clock auction approach. There are other combinatorial auction mechanisms in existence. We have chosen this one for the simplicity it affords to the bidder in putting winning packages together and for the way that it allows bidders to quickly discover prices in a complex market. The combinatorial clock auction operates in two main phases, the price discovery (clock) phase and the final assignment phase. These are now described in turn.

### A. Price Discovery Phase

To begin, the auctioneer has a set of bands in different locations, $\{B_1, B_2, ..., B_m\}$, to sell. A band

---

[1] For clarity it should be noted that throughout this paper *band* refers to a swathe of spectrum for sale such as the 800MHz *band* or 2.6GHz *band*. *Channels* and *channelizations* refer to the individual assignments requested by, or made available to, bidders.

[2] Though an example of an auction that supports asymmetric bids exists [2].

$B_j$ is auctioned for a limited period of time $\tau$, and the auction covers $T$ periods. A bidder can ask for a channel in the band $B_j$ for $t' \leq T$ periods. In the following subsections we will denote by $B_{jt}$ the band $B_j$ in period $t$, where $t \in \{1, 2, \ldots, T\}$. A package bid is a combination of channels in different locations, bands and time periods, and the corresponding price, $p_i$, the buyer is willing to pay for it. It is the job of the clock phase to let the market discover the prices for each item on sale. Bidders may AND items in different locations and semesters and may AND or (2-bid) XOR frequency demands for a given time/location.

In the clock phase the auctioneer repeatedly posts fixed prices for each item, or class of items in a multiple unit auction, on sale to the bidders in a series of rounds. Each category, or unique set of items, has its own "clock price". So, for a particular band of spectrum, for a particular semester, in a particular geographic location there is a clock which represents the price of that bundle. The term clock is used as the price is monotonically ascending, i.e. it can only be ticked upwards during the clock phase of the auction. In each round the bidders can indicate that they will buy packages of the items at the prices indicated. It is this aspect that simplifies the bidding process for the buyer; it simply has to select items whose price is lower than its private valuation and submit its package bid to the auctioneer. If the auctioneer detects excess demand for any item after a round of bidding has closed, it increases the posted price for that item and opens another round of bidding. Again, the bidders can indicate that they will buy packages of the items at the, possibly new and higher, prices indicated. During the clock phase the bidders discover prices at/near the competitive equilibrium, i.e. prices that roughly match supply with demand. In fact the clock price discovery phase approximates a second-price auction. We have modeled our implementation of the clock price discovery phase in our auction on that proposed by Porter et al. [6].

Determining the presence of excess demand in a complex combinatorial setting, especially one which facilitates XOR bids, albeit limited, is not trivial. However, even somewhat crude approaches can be used to detect excess and raise the clock prices. The clock phase of the auction ends when all excess demand is removed from the market; in an ideal situation the prices would stop rising when supply exactly equals demand. However, in complex multi-item-unit, multi-item-type auctions it is unlikely that this will occur. Consequently, the approach used in this part of the auction may result in the oversupply of certain items at the new (higher) prices, i.e. bidders may not bid for certain items at the new (higher) price. This will happen if their private valuation of the spectrum is lower than the demanded clock price. If this situation arises then the CCA calls for the bids to be assigned using a revenue maximizing approach, i.e. using a winner determination algorithm to determine which combination of the bids that that stood at the last clock price which caused excess demand will maximise its revenue. In the assignment stage, the actual allocation of frequencies for each band/location/time period is determined. Once the winning bids are determined, many possible allocations are possible.

### *B. Winner Determination Problem*

The model for the winner determination problem (WDP) has to take into account not only the availability of spectrum, but also the requirements connected to the allocation of paired and unpaired spectrum in a certain band. In particular, a constraint on the duplex spacing for paired spectrum is required to avoid harmful self-interference. The model presented in this subsection builds on the work presented in [1].

Let us denote by $C_{it} = \{c_{i1t}, c_{i2t}, ..., c_{imt}\}$ the set of channels requested in the i-th bid, where $c_{ijt}$ is the amount of spectrum in the j-th band/location and at period $t$. No limitations are imposed on the bid packaging, and a bidder can ask for both paired and unpaired spectrum in each band. To guarantee that the duplex spacing constraint is satisfied, $c_{ijt}$ has to be specified in terms of the amount of unpaired ($c^{un}_{ijt}$), uplink ($c^{ul}_{ijt}$) and downlink spectrum ($c^{dl}_{ijt}$) that is requested for each band/location/ period.

The WDP can be formulated as follows:

$$\begin{aligned}
&\max \sum_{i=1}^{n} p_i x_i \\
&\text{s.t.} \\
&\sum_{i=1}^{n} c_{ijt} x_i \leq B_j, && \forall j \in \{1,2,...,m\}, \forall t \in \{1,2,...,T\} \\
&\sum_{i=1}^{n} c^{dl}_{ijt} x_i - B_j \leq -\delta_j, && \forall j \in \{1,2,...,m\}, \forall t \in \{1,2,...,T\} \\
&\sum_{i \in S_l} x_i \leq 1, && \forall l \in \{1,2,...,p\} \\
&x_i \in \{0,1\}, && (1)
\end{aligned}$$

where $x_i = 1$ if the i-th bid is accepted, $\delta_j$ is the minimum duplex distance between paired spectrum channels, and $S_l$ is the set of indices corresponding to the package bids submitted by the l-th bidder where at most one package can be accepted (XOR constraint). It follows that $\bigcup_{l=1}^{p} S_l = \{1, 2, \ldots, n\}$, where $p$ is the total number of bidders.

The first constraint in (1) guarantees that the allocated spectrum does not exceed the available spectrum for each band/location/period. To simplify the notation, we will omit in the following the subscripts $j$ and $t$. If the assignment stage is performed as explained in section III-A and if we assume that $c_i^{dl} \geq c_i^{ul}, \forall i \in \{1, \ldots, n\}$, the second constraint in (1) ensures a minimum distance $\delta$ between paired spectrum channels. For paired spectrum channels $i$, the distance between uplink and downlink band is:

$$d_i = d_i(c_i^{ul}, c_i^{dl}) = c_{i-1}^{dl} + c_{i-2}^{dl} + \ldots + c_1^{dl} + U + R + c_k^{ul} + \ldots + c_i^{ul} \tag{2}$$

where $U$ is the amount of allocated unpaired spectrum, $R$ is the amount of unallocated residual spectrum[3] and $k$ is the number of allocated paired channels. As $c_{i-1}^{ul} \leq c_{i-1}^{dl}$, $d_{i-1} \leq d_i \forall i \in \{1, \ldots, n\}$. Therefore, the minimum distance between paired channels is $d_1 = U + R + c_k^{ul} + \ldots + c_1^{ul}$; after simple manipulations it can be seen that $d_1$ corresponds to the opposite of the right hand-side of the second constraint in (1). As $d_1$ is the total amount of residual spectrum, allocated unpaired spectrum and allocated uplink spectrum, this constraint is independent on the frequency assignment order of both paired and unpaired spectrum, as long as the unpaired and residual spectrum are allocated in the centre of the band. In other words, the WDP (1) can guarantee a minimum duplex spacing without dealing explicitly with the assignment of frequencies. This is of the utmost importance as it allows the auction of generic spectrum channels: the outcome of the auction determines which portion of the spectrum is allocated to paired and unpaired spectrum.

It is well known that problem (1) is NP-complete. Although the WDP is formulated as a branch-on-bids search problem, the specialized search formulation proposed in [7], which is exponential with respect to the number of items but polynomial in the number of bids, cannot be used as in this case the good to be sold is not a collection of discrete items. Even fixing the size of the base unit of spectrum, i.e. allowing bids on restricted bundles, would not allow the use of the search algorithm in [7], or similar, as the resulting problem is a multi-unit combinatorial auction. Some instances of (1) may require too long to get the optimal solution. However, if an anytime search algorithm is adopted, a feasible solution can be obtained by terminating the algorithm after a certain amount of time.

### C. Carrier Aggregation Model

Let us start with an example. Suppose the auctioneer has 2 bands of 50 MHz each to sell. Suppose also that 5 bidders submit 5 package bids of 20 MHz each. Although the amount of available spectrum is sufficient to accommodate all the submitted bids, only 4 of them can be accepted. This outcome results from the partially outdated assumption that assigned channels should be contiguous.

In recent years the development of frequency-agile radios capable of taking advantage of non-contiguous frequency bands has opened up new opportunities for flexible use of radio spectrum. We argue that this increased flexibility should be taken into account in order to design more efficient auctions.

On the basis of these observations, we modeled an auction mechanism that allows bidders to request combinations of contiguous and non-contiguous channels. In particular, each package bid $i$ contains a combination of $c_i$ requests for contiguous spectrum and $\overline{c}_i$ requests for non-contiguous spectrum. To simplify the discussion, in the following we drop the dependence on time of the bids. However, the proposed model can be easily extended to include the time dimension. For each request $k$ of contiguous spectrum ($1 \leq k \leq c_i$) the package bid $i$ specifies the amount of requested spectrum $b_{ik}$ and the set of bands $C_k$ where the requested spectrum should be allocated. For each request $k$ of non-contiguous spectrum ($1 \leq k \leq \overline{c}_i$) the package bid $i$ specifies the amount of requested spectrum $\overline{b}_{ik}$ and the set of bands $\overline{C}_k$ where the requested spectrum should be allocated. Moreover the bidder can specify that the requested spectrum should be allocated in at most $q_{ik}$ non-contiguous channels ($q_{ik} \leq |\overline{C}_k|$), i.e. $q_{ik}$ is the maximum number of non-contiguous channels that the bidder is willing to accept. This means that $f_{ik} = \frac{\overline{b}_{ik}}{q_{ik}}$ is the minimum channel that the bidder is willing to accept. The resulting WDP is formalized in (3), where $x_i = 1$ if the i-th bid is accepted, $p_i$ is the price that the buyer is willing to pay for the i-th bid, $z_{ikj} = 1$ if the contiguous request $k$ in the i-th bid is allocated in band $j$, $y_{ikj} = 1$ if the non-contiguous request $k$ in the i-th bid is allocated in band $j$, $\overline{z}_{ikj}$ is the amount of spectrum corresponding to the non-contiguous request $k$ in the i-th bid that is allocated in band $j$, and $M$ is a large constant.

Constraint (3a) guarantees that the allocated spectrum does not exceed the available spectrum $B_j$ for each band. Constraint (3b) ensures that only one channel is allocated for each contiguous request $k$ in the i-th bid. Likewise constraint (3c) ensures that the amount of spectrum allocated for each non-contiguous request $k$ in the i-th bid is equal to the

---

[3] The allocation resulting from the solution of the WDP does not guarantee that all the spectrum is assigned. For each band the unallocated residual spectrum is $R_{jt} = B_{jt} - \sum_{i=1}^{n} c_{ijt} x_i$.

$$\max \sum_{i=1}^{n} p_i x_i$$

s.t.

$$\sum_{i=1}^{n}\sum_{k=1}^{c_i} b_{ik} z_{ikj} + \sum_{i=1}^{n}\sum_{k=1}^{\overline{c}_i} \overline{z}_{ikj} \le B_j, \qquad \forall j \in \{1,2,...,m\} \tag{3a}$$

$$x_i - \sum_{j \in C_k} z_{ikj} = 0, \qquad \forall i \in \{1,2,...,n\}, \forall k \in \{1,2,...,c_i\} \tag{3b}$$

$$\overline{b}_{ik} x_i - \sum_{j \in \overline{C}_k} \overline{z}_{ikj} = 0, \qquad \forall i \in \{1,2,...,n\}, \forall k \in \{1,2,...,\overline{c}_i\} \tag{3c}$$

$$\overline{z}_{ikj} \ge f_{ik} - (1 - y_{ikj})M, \qquad \forall i \in \{1,2,...,n\}, \forall k \in \{1,2,...,\overline{c}_i\}, \forall j \in \overline{C}_k \tag{3d}$$

$$\overline{z}_{ikj} \le y_{ikj} M, \qquad \forall i \in \{1,2,...,n\}, \forall k \in \{1,2,...,\overline{c}_i\}, \forall j \in \overline{C}_k \tag{3e}$$

$$z_{ikj} \in \{0,1\}, \qquad \forall i \in \{1,2,...,n\}, \forall k \in \{1,2,...,\overline{c}_i\}, \forall j \in C_k$$

$$y_{ikj} \in \{0,1\}, \overline{z}_{ikj} \in [0, \overline{b}_{ik}] \qquad \forall i \in \{1,2,...,n\}, \forall k \in \{1,2,...,\overline{c}_i\}, \forall j \in \overline{C}_k$$

$$x_i \in \{0,1\} \qquad \forall i \in \{1,2,...,n\}$$

requested channel $\overline{b}_{ik}$. Finally the requirement on the maximum number of non-contiguous channels is modeled in (3d) and (3e).

## IV. RESULTS AND DISCUSSION

In order to assess the effects of the auction mechanism described in Section III, we carried out a wide range of simulations of different scenarios. Our first set of results are based on a scenario which is drawn from a real auction format designed on behalf of Ofcom [8]. We then go on to look at increasingly flexible scenarios and expressive bids. We vary the duplex spacing, vary the valuations the bidders make on FDD and TDD spectrum, allow for asymmetric FDD bids and allow for asks that include AND and XOR. As will be seen from the results that follow the auction is able to handle this level of sophistication. The results also illustrate how effective the auction process is at assigning spectrum as well as tease out the impact of specific factors. For each scenario presented, $10^3$ random instances were simulated and average results are presented.

### *A. Variable Basic Bid Blocks*

The first aspect of the auction that we evaluated was the effect of enabling the bidder to specify the amount of spectrum it wanted, rather than forcing the bidder to accept lots defined by the auctioneer. As discussed in Section III the notion of flexibility in an auction pertains to the ability of the bidders to ask for the items that they *actually want* and expressiveness pertains to their ability to describe packages of such items that suit their needs. In the scenarios described here, the results of which are summarized in Table I, an auction for 2 bands of spectrum was simulated. These two bands correspond to the 800MHz and 2.6GHz bands of spectrum that are currently being auctioned by regulators for probable LTE or WiMAX use [8].

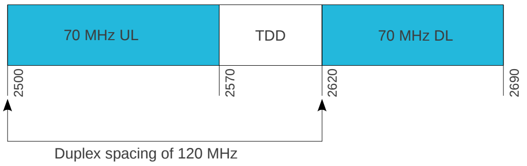

Fig. 1. The proposed allocation of spectrum to FDD and TDD use in the 2.6GHz band [8].

The 800MHz band consists of 81MHz from 791Mhz to 862Mhz, only 70MHz of which is on offer. Ofcom, as with most other regulators [2], propose to package the sale of this spectrum for paired FDD use only, i.e. 30MHz FDD uplink from $791-821$MHz and 30MHz FDD downlink from $832-862$MHz. 11MHz of spectrum between these paired bands was not be auctioned concurrently with the paired spectrum. Bidders could ask for $2x5$MHz or $2x10$MHz packages in these bands.

The 2.6GHz band consists of 190MHz from 2500MHz to 2690MHz. This spectrum is to be offered for both paired and unpaired use, as depicted in Figure 1. A duplex spacing requirement of 120MHz was determined for this band; this duplex spacing implies that the maximum allocation of spectrum for FDD use in this band is 140Mhz, i.e. 2x(190MHz-120MHz).

The first scenario, detailed in Table I(a), reflects the bid ask rules currently set out by Ofcom [8]. As with all of the simulations presented in this paper, 100 bidders, with individual profiles, were generated for each scenario instance. In scenario (a) bidders in the 800MHz band were allowed to bid for either

2x5MHz or 2x10MHz packages of FDD spectrum. 2.6GHz bidders could either bid for a 2x10Mhz package of FDD spectrum or the single 50MHz TDD slot. Corresponding to the Ofcom approach, in scenario (a) only OR bids were allowed, i.e. the bidders only bid for 800MHz or 2.6GHz spectrum. In scenarios (b) and (c) bidders could make XOR package bids for both bands [4]. Furthermore, the private spectrum valuations for all bidders in this subsection, regardless of the band or type of spectrum they sought, were drawn from the same range.

In order to investigate the effect of allowing variable bid sizes each bidder was assigned two spectrum demand profiles; a real demand profile and a forced demand profile. The real demand corresponds to the actual amount of spectrum that would suffice for the bidder given its own internal radio constraints and capacity demands. The forced demand corresponds to the amount of spectrum that a bidder would actually have to bid for given the auction format, i.e. the minimum lot size.

Firstly, each bidder was assigned a real or *actual* spectrum demand which was drawn from the real interval corresponding to bid sizes on offer. As the 800MHz FDD spectrum was offered in either 2x5MHz or 2x10MHz lots, bidders were therefore assigned an *actual* demand in the range 2x[0,10]. So, if a bidder was firstly assigned a demand of 2x4.55MHz it would then be forced to bid for 2x5MHz using the auction rules for scenario (a). Consequently, while the auction managed to assign 99.86% of the spectrum in this scenario, this assignment only represented a real, or actual, demand of 73.32% of the spectrum sold. Put another way we can say that 26.68% of the spectrum available is wasted through the assignment process.

TABLE I
FLEXIBLE SCENARIOS (B) AND (C) VERSUS MORE STANDARD APPROACH (A)

| | Assignment Efficiency (%) | Wasted Spectrum (%) | Permitted 800MHz Asks | Permitted 2.6GHz Asks |
|---|---|---|---|---|
| a | 99.86 | 26.68 | F[5:5:10] | F[10], T[50] |
| b | 99.98 | 3.53 | F[1:1:15] | F[1:1:10],<br>T[1:1:50] |
| c | 99.99 | 0 | F[0,15] | F[0,10],<br>T[0,50] |

In scenarios (b) and (c) we introduce flexibility to the auction by relaxing the restriction on the bids that may be submitted. Whereas most, if not all, regulators discretize the spectrum offering into 5MHz or 10MHz blocks, in scenario (b) bidders can ask for bids in terms of 1MHz blocks. In this scenario, a bidder whose real spectrum demand is 38.5MHz of 2.6GHz TDD spectrum is only forced by the auction format to increase its submitted bid to 39MHz. The *wasted spectrum* drops to 3.53% in this case. In scenario (c) all restrictions on basic bid blocks are eliminated, i.e. bidders can ask for their exact real spectrum demands, within the caps indicated in Table I. It can be seen that in this scenario the assignment efficiency is at 99.99% and there is no wasted spectrum.

### *B. Variable Duplex Spacing, Variable FDD and TDD spectrum valuations*

The next set of results shown in Table II are based on scenarios in which two key elements are varied. First of all the bidders in the scenarios shown place different valuations on TDD spectrum and FDD spectrum as indicated in the column labelled F($):T($). Secondly the minimum duplex spacing is varied. In all cases the bidders can request FDD spectrum in symmetric lots of $2x$ 5MHz, 10MHz or 15MHz and TDD spectrum in lots of 10MHz or 20MHz. For the sake of clarity and conciseness in this scenario a single block of 200MHz is being auctioned. The parameters used in this auction scenario do not extrapolate from any existing spectrum auction unlike those used in the previous subsection.

The pattern that emerges is as follows: If the TDD bidders value the spectrum more than the FDD bidders, then all spectrum ends up being allocated to TDD bidders as would be expected. This is the case in scenarios (b), (d) and (f). In the case where the opposite is true the bulk of assignments go to the FDD bidders as can be seen in scenarios (c), (e) and (g). However because a duplex spacing is required some spectrum may still be assigned to the TDD bidders. *The amount of spectrum that gets assigned to TDD bidders depends on how big the duplex spacing is.* A duplex spacing of 100MHz means that all spectrum can be assigned to the FDD bidders. However bigger duplex spacings result in some spectrum being set aside for TDD despite these bidders valuing the spectrum less.

### *C. Variable Asymmetric FDD Assignments*

The next aspect of the auction mechanism that allows for freer expression on the part of the bidders is the ability of FDD bidders to ask for asymmetric assignments of paired uplink and downlink spectrum. In the simulations presented in this section, the results of which are summarized in Table III, bidders may ask for UL and DL assignments in asymmetric

---

[4] The notation in the *Permitted 2.6GHz Asks* column of Table I for scenarios (b) and (c) indicates that bidders' asks can be assigned in the range those intervals. In scenario (b) the intervals have discrete steps. In scenario (c) the intervals are continuous.

TABLE III
THE RESULTS OF ALSO ALLOWING FOR VARIABLE ASYMMETRIC FDD REQUESTS

| | Assignment Efficiency (%) | UL FDD | TDD | DL FDD | Duplex Gap | Asymmetry Bid Range | Resulting UL:DL Split | Permitted Asks |
|---|---|---|---|---|---|---|---|---|
| a | 74.99 | 49.99 | X | 99.99 | 100 | DL = UL x 2 | 1:2 | F[5:5:15] |
| b | 86.84 | 73 | X | 99.74 | 100 | DL = UL x [1:0.1:2] | 1:1.36 | F[5:5:15] |
| c | 99.095 | 39 | 81.19 | 78 | 100 | DL = UL x 2 | 1:2 | F[5:5:15], T[10:10:20] |
| d | 99.54 | 46.51 | 83.36 | 69.23 | 100 | DL = UL x [1:0.1:2] | 1:1.48 | F[5:5:15], T[10:10:20] |
| e | 98.15 | 24.55 | 122.642 | 49.115 | 134 | DL = UL x 2 | 1:2 | F[5,15], T[15,45] |
| f | 99.27 | 30.55 | 122.40 | 45.59 | 134 | DL = UL x [1:0.1:2] | 1:1.49 | F[5,15], T[15,45] |

TABLE II
THE RESULTS OF VARYING THE FDD/TDD SPECTRUM ALLOCATIONS AND VARYING THE DUPLEX GAP

| | Ass. Eff.(%) | UL FDD | TDD | DL FDD | Dup. Gap | F($):T($) |
|---|---|---|---|---|---|---|
| a | 100 | 56.05 | 87.90 | 56.05 | 100 | F($)=T($) |
| b | 100 | 0 | 200 | 0 | 100 | F($)<T($) |
| c | 100 | 99.98 | 0.04 | 99.98 | 100 | F($)>T($) |
| d | 100 | 0 | 200 | 0 | 134 | F($)<T($) |
| e | 99.96 | 65.98 | 68 | 65.98 | 134 | F($)>T($) |
| f | 99.96 | 0 | 200 | 0 | 150 | F($)<T($) |
| g | 100 | 50 | 100 | 50 | 150 | F($)>T($) |

ratios up to a limit where the amount of DL spectrum is twice the UL spectrum requested.

The notation in Table III is as before but for the addition of a column marked *Asymmetry Bid Range* which indicates the kind of asymmetric requests made by the FDD bidders. Two types of asymmetry were modelled. Firstly, in scenarios (a), (c) and (e) all bidders asked for asymmetric UL and DL assignments in the ratio of 1:2. Secondly, in the remaining scenarios, (b), (d) and (f), bidders were assigned an asymmetric DL demand drawn from the interval $[1 : 0.1 : 2]$, i.e. the asymmetry they demanded could range from symmetric up to a $1 : 2$ split. Again, a 200MHz band was simulated and all bidders, whether asking for FDD or TDD assignments, were assigned spectrum valuations from the same range of valuations so there was no valuation bias in favour of either FDD or TDD allocations.

For scenarios (a) and (b) we did not allow bidders to ask for TDD spectrum in order to see the effect of asymmetric assignments without the constraint of having to allocate spectrum between FDD and TDD use. In (a) the FDD assignments break down exactly in line with the demands, as indicated in the *Resulting UL:Dl Split* column. The duplex spacing of 100MHz means that 50MHZ of spectrum cannot be assigned to FDD users when they ask for spectrum in the $1 : 2$ ratio. In scenario (b) the asymmetry of the request varies from $1 : 1$ to $1 : 2$. In this case the auctioneer is able to assign more spectrum, and thereby maximize its revenue, by choosing from the winning bids those with the more symmetrical bids; the uplink to downlink allocation ratio here is $1 : 1.36$ In this case it can achieve a $86.84\%$ assignment efficiency.

Scenarios (c) to (f) demonstrate the effects of varying the duplex spacing and also varying the size of the basic bid blocks that the bidders can bundle together. It can be seen that in all cases where the FDD asymmetry is fixed at 1:2, the auctioneer assigns the FDD in that ratio. However, in the scenarios where the bidders have made bids in the range [1:0.1:2], the auctioneer assigns bids broadly in line with the average bidder asymmetry, i.e. 1:1.5. The presence of TDD bids reduces the impact of the duplex gap constraint, i.e. it enables the most valuable FDD requests to emerge through the clock process and be selected by the WDP algorithm, as an appropriate mix of FDD and TDD bids can be found to both respect the duplex gap *and* maximise revenue. This is reflected in the average resulting UL:DL split ratio rising from $1 : 1.36$ (b) to $1 : 1.49$ (f).

### *D. Variable Asks, Multiple Bands, AND/XOR Packages*

The results presented in this section correspond to scenarios in which multiple bands of spectrum were auctioned using all of the flexibility described in the previous subsection. While the main WDP algorithm, (1), has been evaluated for auction scenarios incorporating the combinatorial assignment of spectrum across 5 bands, for the sake of clarity we only present results from a set of $2-$band scenarios; Band A, a block of 300MHz spectrum, and Band B, a block of 200MHz spectrum. Table IV, summarizes the results of simulations in which we evaluated the auction system subject to the variation of the bid ask sizes, an imposed duplex gap of 100MHz and asymmetry of FDD requests (values drawn from $[1 : .25 : 2]$).

TABLE IV
RESULTS OF AUCTION SCENARIOS INVOLVING AND/XOR VARIABLE PACKAGE BIDS ACROSS 2 BANDS

| | Total Eff. (%) | Band A Eff. (%) | Band A FDD | Band A TDD | Bid Type | Band B Eff. (%) | Band B FDD | Band B TDD |
|---|---|---|---|---|---|---|---|---|
| a | 97.3 | 95.66 | 220.03 | 66.97 | AND | 99.77 | 96.05 | 103.49 |
| b | 97.70 | 99.51 | 199.85 | 98.96 | XOR | 100 | 122.82 | 77.18 |
| c | 99.72 | 99.54 | 245.35 | 53.29 | AND/ XOR | 100 | 148.26 | 51.74 |

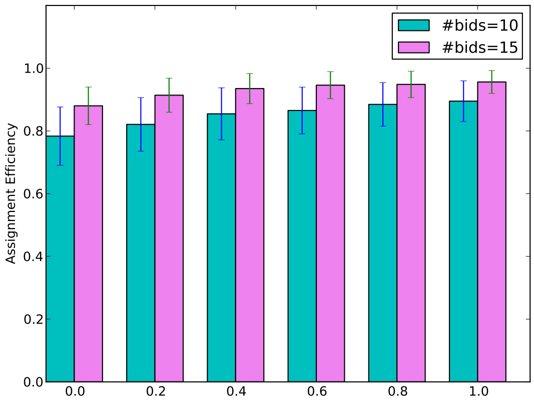

Fig. 2. Assignment efficiency with respect to the fraction of non-contiguous requests.

Furthermore, as indicated in the *Bid Type* column of Table IV, bidders were able to make package bids using either AND or XOR across the two bands for auction. The results indicate that the auction mechanism continues to achieve a high assignment efficiency; the results are in line with those presented in the previous subsections.

### *E. Variable InterBand Carrier Aggregation Allocation*

Finally, we investigated the flexibility of the model proposed in section III-C by modeling the spectrum auction for proposed LTE-Advanced networks. We chose this particular scenario because of the high spectrum flexibility supported by LTE-Advanced systems. In particular, the possibility of inter-band carrier aggregation is one of the key features of LTE-Advanced. In our simulations we considered 5 bands destined to IMT-Advanced technologies (790-862 MHz, 88-960 MHz, 1710-2170 MHz, 2300 - 2400 MHz and 2500-2690 MHz). Given their characteristics, we consider the first two bands on the list to be substitutive of each other (sub 1GHz spectrum) and likewise for the last two (higher frequencies).

We ran $10^3$ random instances of (3) for $n \in \{10, 15, 30\}$ bids. For each instance a bid is generated so as to contain a combination of channels in the sub 1GHz spectrum, in the typical cellular spectrum and in the higher frequencies. We analysed the effect of the flexibility introduced by the model described in section III-C by varying the percentage of non-contiguous spectrum requests along substitutive bands. Figure 2 shows the assignment efficiency with respect to the percentage of non-contiguous requests. The assignment efficiency increases with the number of bids containing requests for non-contiguous spectrum. The case of total inflexibility of the bidders, i.e. bidders only request contiguous channels, corresponds to a lower assignment efficiency. It is interesting to note that the same trend is exhibited independently of the number of bids.

## V. CONCLUSIONS

Spectrum auctions continue to move in the direction of market-based assignment mechanisms. However the design of auctions to-date embody much of the traditional thinking about how spectrum is managed. In this paper we have shown that the parameters of the auction process reflect what can be considered to be traditional notions of spectrum allocation and conserve the role of the regulator as the central decision maker. We have offered an alternative approach that places more of the decisions in the hands of the bidders and allows the market to drive the *allocation* and *assignment* of spectrum. Our flexible auctions lead to emergent market-driven FDD and TDD allocations and are expressive enough to allow bidders to ask for what they really want. No restriction is imposed on the size of the basic bid blocks. Duplex spacing is included in the auction model, thus eliminating the need for band planning. The amount of spectrum allocated to FDD and TDD is decided based on the bidders' demands. Asymmetric requests of paired uplink and downlink spectrum are supported. Non-contiguous spectrum requests are modeled to keep up with the development in interband carrier aggregation. The increased freedom in the proposed auction mechanisms result in an improved spectrum allocation efficiency. Moreover some of the decision variables currently chosen by the auctioneer through consultation emerge as outcomes of the market demands.

There is much more work to be done in evaluating this kind of auction from a a technical, regulatory and economic perspective. The next steps will involve exploring how any residual spectrum is allocated and assigned as well as extending the work on carrier aggregation. The auctions described here can be used on a frequent basis as distinct from infrequently for longer term assignments and we will also explore this avenue. There is also a body of work to be carried out on the bidder side in investigating how easy it is for bidders to compose bids and understand options in what is now a multi-dimensional landscape.

## ACKNOWLEDGEMENTS

This material is based upon works supported by the Science Foundation Ireland under Grant No. 10/CE/I1853. It has also been supported by funding from the European Community's Seventh Framework Programme (FP7-ICT-2009-4) under grant agreement number ICT-248560 (COGEU).

## REFERENCES

[1] T. Forde, I. Macaluso, and L. Doyle, "Exclusive sharing & virtualization of the cellular network," in *New Frontiers in Dynamic Spectrum Access Networks (DySPAN), 2011 IEEE Symposium on*. IEEE, 2011, pp. 337–348.

[2] DotEcon (Marsden, R. and Sexton, E. and Siong, A.), "Fixed or flexible? A survey of 2.6GHz spectrum awards," UK, 2010.

[3] H. Falaki, D. Lymberopoulos, R. Mahajan, S. Kandula, and D. Estrin, "A first look at traffic on smartphones," in *Proceedings of the 10th annual conference on Internet measurement*, ser. IMC '10. New York, NY, USA: ACM, 2010, pp. 281–287.

[4] "CEPT Report 23, Complementary Report to Report B (CEPT Report 22) from CEPT to the European Commission in response to the Mandate on: "Technical considerations regarding harmonisation options for the Digital Dividend" "Technical Options for the Use of a Harmonised Sub-Band in the Band 470 - 862 MHz for Fixed/Mobile Application (including Uplinks)"," 2007.

[5] T. Sandholm, ""Expressive Commerce and Its Application to Sourcing: How We Conducted $25 Billion of Generalized Combinatorial Auctions," Artificial Intelligence Magazine, vol. 28, no.3, 2007, AAAI Press."

[6] D. Porter, S. Rassenti, A. Roopnarine, and V. Smith, "Combinatorial auction design," *Proceedings of the National Academy of Sciences of the United States of America*, vol. 100, no. 19, p. 11153, 2003.

[7] T. Sandholm and S. Suri, "BOB: Improved winner determination in combinatorial auctions and generalizations," *Artificial Intelligence*, vol. 145, no. 1-2, pp. 33–58, 2003.

[8] Ofcom, "Consultation on assessment of future mobile competition and proposals for the award of 800 MHz and 2.6 GHz spectrum and related issues," UK, 2011.