Incentive Mechanisms based on Minority Games in Heterogeneous Delay Tolerant Networks

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Abstract—In this paper we design an incentive mechanism for heterogeneous Delay Tolerant Networks (DTNs). The proposed mechanism tackles a core problem of such systems: how to induce coordination of DTN relays in order to achieve a target performance figure, e.g., delivery probability or end-to-end delay, under a given constraint in term of network resources, e.g., number of active nodes or energy consumption. Also, we account for the realistic case when the cost for taking part in the forwarding process varies with the devices’ technology or the users’ habits. Finally, the scheme is truly applicable to DTNs since it works with no need for end-to-end connectivity.

In this context, we first introduce the basic coordination mechanism leveraging the notion of a Minority Game. In this game, relays compete to be in the population minority and their utility is defined in combination with a rewarding mechanism. The rewards in turn configure as a control by which the network operator controls the desired operating point for the DTN. To this aim, we provide a full characterization of the equilibria of the game in the case of heterogeneous DTNs. Finally, a learning algorithm based on stochastic approximations provably drives the system to the equilibrium solution without requiring perfect state information at relay nodes or at the source node and without using end-to-end communications to implement the rewarding scheme. We provide extensive numerical results to validate the proposed scheme.

Index Terms—Minority Game, Energy Efficiency, Heterogeneous Delay Tolerant Networks, Nash equilibria, Learning algorithms, Mechanism Design.

I. INTRODUCTION

Delay Tolerant Networks (DTNs) are designed to sustain communications with no need for persistent connectivity. As such, they configure as a mean to exchange data without the help of dedicated network infrastructure. As described in related literature [9], [14], the way to overcome disconnections in such systems is by message replication. The replication mechanism resembles an epidemic where the infection is represented by the data generated by a source node and the infected nodes are those which fetch data from peer nodes.

In practice, DTNs composed by mobile devices could include smartphones, tablets or other devices using multiple wireless interfaces. For such devices, OS APIs are available to program dedicated applications for direct data exchange with peer nodes during radio range contacts [13].

Using such a flexible mechanism for communication, DTN networks can use mobile terminals as information carriers: as such, they can be employed to off-load infrastructure networks during critical demand situations. This is the case of modern flashcrowds or similar events: they represent a peculiar novel type of hotspot scenario were infrastructured networks are overused. In these events, due to the high densities of user-generated contents related to a specific geographic location, the operator faces massive demand on the limited spectrum of infrastructure networks, therefore leading to deteriorating wireless quality for all subscribers. The solution that is classically adopted by operators in this context is to deploy contingency sites to increase coverage, with the incurred costs for installation, removal and overall network engineering for interference mitigation.

These contingency solutions are designed for specific events: in that scenario, the appeal of DTNs is precisely that they are flexible in that users act directly as relays thus contributing to reducing infrastructure costs while ensuring good coverage. Furthermore, users can keep data exchange local, e.g., event-related information can be disseminate immediately among the participants with no need to outreach remote servers. This architecture is thus expected to reduce network costs, while offering a new service that may interest a large community of users.

In order to make a DTN for local data exchange truly operational, several technical issues need to be tackled in order. To date, the most attention from the research community has been attracted on how to route messages reliably towards the intended destination(s). Replication of the original message by the so called epidemic routing protocol ensures that at least some copy will reach the destination node with high probability minimizing the delay to reach the intended destinations. In turn, the standard optimization problem becomes how to maximize the delivery probability under constraints on the resources spent to forward it to the destination. Several papers have further extended the possible optimization to the use of activation and/or forwarding control at relays [11].

In this paper we rather focus on a fundamental aspect that is usually overlooked in DTN literature, namely, the autonomous activation problem. DTN performance optimization is usually made under the implicit assumption that relays are willing to cooperate with the source node. But, core point is whether owners of relay devices are willing to have battery depleted to sustain DTNs communications. In turn, massive de-activation of relays becomes a core threat which hinders any possible attempt to optimize network performance.

To this respect, a fundamental observation is that relays may have heterogeneous energy requirements and so does the cost for activation they experience [4], [8]. In fact, there are two main sources of heterogeneity that we need to model. First,
DTN nodes may belong to different categories, e.g., PDA, laptop, mobile and/or have related communication/energy-autonomy features. Those are determined by transmission range, mobility, memory, energy capacity and active radio interface such as WiFi and Bluetooth. Second, heterogeneity also includes differences in behavior of devices’ owners[12]. This covers a realistic scenario in which the cost experienced by users to activate is related to their activities and the priority they give to tasks other than relay operations, e.g., facebook updates or phone calls may be prioritized over relaying activities depending on the user profile.

**Contribution of the paper:** The main contribution proposed lies in the design of a credit-based mechanism for relay participation in DTNs. Our mechanism design attains a twofold objective. First, the decision to participate to relaying or not is taken autonomously by relays according to the incentive scheme, i.e., the choice to activate or not depends on the local decision taken by the device and no centralized signal exchange is required. Second, since incentives engender a competition among relays that play strategies on their activation, they can be driven to attain a desired operating point for the DTN. Such an operating point, in turn, is precisely the solution of a joint optimization problem involving the number of active relays.

Thus, the reward offered is a control variable by which the source can tune the number of relays taking part in the forwarding process. To do so, we define a specific utility structure as linear combination of the success probability of a relay and the energy cost for activation. The success of a tagged relay depends on the number of active opponents met: the bigger the number of relays participating to the message delivery, the higher the delivery probability for the message, but indeed the less the chance for the tagged relay to receive a reward from the system. Furthermore, we will consider different information scenarios: message sources may have different information about the energy cost of relays. In the first scenario, the heterogeneity of energy is based on the type of device (e.g., tablet, iPad, smartphone, laptop, access point). Sources can identify the type of devices at each contact, thus the game is played under complete information on the energy cost of each class. In the second scenario, we associate the energy cost to the activity of devices’ owners or their profile.1

More in detail, our approach is grounded into the theory of the Minority Game (MG) [10] which tunes performance of competing relays and welfare of the DTN (number of message copies and delivery message). We thoroughly investigate the properties of our coordination game in which relays compete to be in the population minority.

**Remark 1:** A peculiar requirement for our credit exchange mechanism is that it does not require end-to-end communications. Due to the fact that feedback messages in DTNs may incur into large delays, in fact, the exchange of credits between relays should not require feedback messages. In order to overcome lack of feedback, our solution adopts a learning algorithm based on stochastic approximations. This algorithm performs in distributed fashion and operates based on local estimates performed by relays. As such, it is fully decentralized with no need to reconstruct full state information neither at the source nor at relays.

The remainder of this paper is organized as follows. The next section introduces the system model. Results for the equilibria of the MG in device-dependent heterogeneous DTNs are derived in in Sec. III. The extension to the incomplete information scenario in user-dependent heterogeneous DTN is provided in Sec. IV. A distributed reinforcement learning algorithm able to drive the system to the desired operating point is derived in Sec. V. Numerical reinforcement learning algorithm for validating the outcomes of the theoretical analysis are reported and discussed in Sec. VI. Deep discussion of assumptions, limitations and future work are provided in Sec. VII. Final remarks are reported in Sec. VIII.

II. NETWORK MODEL

In this section, we present the overall architecture and key insight into our mechanism design.

A. System architecture and reward mechanism

We consider a DTN with several source-destination pairs s_i and N relay nodes. Relay nodes are equipped with a wireless interface allowing communication with other mobiles in their proximity. Messages are generated at the source nodes and need to be delivered to the destination nodes. The network is assumed to be sparse: nodes are isolated with high probability at any time instant.2 Communication opportunities arise whenever two nodes fall within reciprocal radio range, i.e., a “contact” occurs. We assume contacts last enough to ensure the transmission of all data needed for a message relaying. Also, we assume that inter contact times between any pair of nodes are independent identically distributed (i.i.d.) random variables. Note that to this respect our framework generalizes i.i.d. models proposed in literature to mimic synthetic mobility processes, e.g., Random Walk or Random Waypoint, since we allow for general intermeeting distributions, which may well include heavy tailed ones.

A source attempts to deliver a message to destination generating several copies among relays. Each such copy contains a time stamp indicating its age and can be dropped when it becomes irrelevant, e.g., after time \( \tau \). \( \tau \) is also the horizon by which we intend to optimize network performance. Due to lack of permanent connectivity, we exclude the use of feedback that allows the relays or sources to know whether the message has been successfully delivered to its destination or not. For the same reason, the design of our activation mechanism should not require centralized coordination nor full state information and any such scheme should indeed run fully distributed on board of the relay nodes.

1 In view of the reward mechanism proposed, this scenario can also reflect the importance for a tagged user to use the DTN to exchange its own messages since it can act as both a relay and a source.

2 This is also the case when disruption caused by mobility occurs at a fast pace compared to the typical operation time of protocols, e.g., the TPC/IP protocol suite.
Let $g_j$ be the energy cost for relay node $j$ when it remains active during $[0, \tau]$. This cost captures the heterogeneity of nodes in DTN in terms of cost for activation.

Now assume that aim is to achieve a target performance figure (e.g., delivery probability, end-to-end delay). Without loss of generality, we focus on the probability of successful delivery. However, our results extend to any performance measure monotonically increasing with the number of active relays. Given target performance figure and the parameters of the DTN (e.g., mobility, transmission range, density of nodes), it is possible to estimate the number of nodes which should activate, named $\Psi$, in order to guarantee this target figure. Now the question is how to stimulate $\Psi$ user nodes to participate to delivery message in a distributed manner.

To this aim each source $s$ proposes a reward for relays. For instance, the reward can be a certain number of credits that relays may use to send their own messages over the DTN. Also, the reward $r^s_j$ is based on the type of device $j$. In fact, nodes with larger battery capacity might choose to be more active to collect the reward, while nodes with a limited battery capacity may participate less to save energy.\(^3\) In particular, the relay node of type $j$ receives a positive reward $r^s_j$ if and only if it is the first one to deliver the message to the destination.

Overall, sources satisfy performance requirements by activating relays by rewarding: larger rewards engender more nodes to be active which yields higher delivery probability at the expense of battery depletion and network’s lifetime. This trade-off rises the following question: How to define the reward in order to activate enough relay nodes such as to attain the assigned performance figure?

The answer to this question is investigated hereafter.

### B. Network Game

When a message is generated by a source node, competition is engendered by the general incentive mechanism during the message lifetime $\tau$. Each mobile has two strategies: either to participate to forwarding, i.e., pure strategy $transmit (T)$, or not to participate, i.e., pure strategy $silent (S)$. Each strategy corresponds to a certain utility for the relay. Clearly, the payoff of a relay should depend on the actions performed by $N - 1$ opponent mobiles. In our mechanism, the utility is designed such in a way that, for each player in the game, it is worth playing a given action if the number of peer nodes that adopt the same strategy does not exceed $\Psi$, i.e., fraction of the total population of interacting nodes. In fact, mobiles of each class $j$ who take the minority action, within the tagged class, win, whereas the majority loses. To this respect activation threshold $\Psi$ is the minority rule of our game.

Let’s now detail how the minority game develops. Assume target probability of successful delivery $D^s_{suc}:

$$D^s_{suc} \geq D^{th}_{suc}. $$

From the sources’ point of view, the probability of successful delivery of a message is given by

$$D_{suc}^s(\{N_T\}) = 1 - \prod_{k \in N_T} Q^k, $$

where $N_T$ is the set of active relay nodes and $Q^k$ is the probability that node $k$ fails to relay the copy of the message to the destination. Then $1 - Q^k$ is the probability that the tagged node succeeds to relay the copy of the message to the destination within time $\tau$. The expression of $Q^k$ depends on the distribution of the inter-meeting intervals.

The number $\Psi$ of active relays will then define such as that target performance figure $D_{suc}^s(\Psi) = D^{th}_{suc}$.

Formally, let $k_T (k_S)$ be the number of agents selecting strategy $T$ (resp. $S$). A tagged relay playing strategy $T$ is member of the minority if $k_T \leq \Psi$, otherwise it loses; silent agents win as $k_S \leq N - \Psi$. Hence the total reward of an active relay in class $j$ is given by $R_j = \sum_{k} r^s_j P_{suc}^s(T, k, s)$, where $P_{suc}^s(T, k, s)$ is the probability that an active node (of class $j$) receives reward $r^s_j$ from source $s$ when $k$ nodes are active. For the sake of simplicity and clearness we assume that every node has the same probability to meet a source and thus all sources use the same mechanism reward, i.e., $r^s_j = r_j$. Hence the total reward of a relay of type $j$ becomes $R = n_s r_j P_{suc}^s(T, k, s)$, where $n_s$ is the number of sources in the network.

In the rest of the paper, we make a key assumption on function $P_{suc}^s(T, k, s)$ which follows naturally since only the first relay to deliver obtains the reward:

**Assumption A:** $P_{suc}^s(T, k, s)$ is decreasing in the number of active relays $k$.

### III. Heterogeneous Devices-dependent Energy Cost

This section considers the case when the energy cost depends on the class of devices the relays belongs to, let them be, e.g., notebook computers, ebook readers and tablet computers such as the iPad, netbooks or smartphones.

We consider $M$ classes of devices in the DTN. Each class contains $N_j$ relays with $N = \sum_j N_j$. For the sake of clarity, we will often refer to the case $M = 2$. However, all results can be easily extended to hold in general case. Assume that $g_1 > g_2$, i.e., nodes belong to class 1 have higher energy cost than class 2 when active. For example mostly devices such as smartphones have smaller power budgets compared to laptops: in turn, their energy cost is higher. Furthermore the power consumption of the WiFi radio represents a large fraction of the overall power consumed by a small device. Hence, we define the utility of an active relay of class $j$

$$U_j(T, k_T) = n_s r_j P_{suc}^s(T, k_T) - g_j \tau$$

where $r_j$ is the reward designed by sources for class $j$. The utility for a silent node is $U_j(S, k_T) = 0$. Now, we should characterize the number of active relay in each class $j$, named $\Psi_j$, able to guarantee the target (1). $\Psi_j$ is thus the per-class minority of our game for the multi-class scenario.
This threshold can be achieved using the reward mechanism \( r^* = (r^*_j)_{j=1,...,M} \) which should obey the following relation:
\[
\forall 1 \leq j \leq M : \quad n_a r^*_j P_{\text{succ}}(T, \Psi_j) = g_j T 
\]  
(3)

Most of the proofs are quite lengthy and are given in our online technical report [5].

A. Pure Nash Equilibrium

Definition 1: A Nash Equilibrium in pure strategies exists if and only if the following two conditions to be satisfied:
\[
\forall 1 \leq j \leq M : \quad \begin{cases} 
U_j(S, k_T) \geq U_j(T, k_T + 1) & \text{for } U_j(S, k_T - 1) \leq U_j(T, k_T) 
\end{cases} 
\]  
(4)

Actually, the above statement says that no player can improve its utility by unilaterally deviating from the equilibrium. The equilibrium of the multi-class games is as follow

Proposition 1: Let \( r \) be the reward mechanism designed by sources. There exists a pure Nash equilibrium. Further, the number of active relay nodes for class \( j \), named \( k_{T,j} \), is the same under all pure Nash equilibria where \( k_{T,j} \) is solution of
\[
\forall 1 \leq j \leq M : \quad r_j P_{\text{succ}}(T, k_{T,j}) = g_j T 
\]  
(5)

Proof: See the full version of the paper [5].

We note that under reward mechanism \( r \) in (5), there are \( \sum_{j=1}^{M} N_j \) pure Nash equilibria. Further, under reward mechanism \( r \) defined in (3), the number of active relay nodes in each class at a pure Nash equilibrium, is \( \Psi_j \), with \( \Psi = \sum_{j=1}^{M} \Psi_j \). This corresponds to the target of sources in the system. Unfortunately the pure equilibrium could fail to achieve a certain fairness between node relays since only a part of relay nodes participate to relaying messages. To overcome this problem, we use another concept of equilibrium, named full mixed equilibrium, in which a relay node will be active only for a fraction of the time. We also design a learning algorithm that allows the system to converge to a mixed equilibrium.

B. Mixed Nash Equilibrium

Let`s consider now that relay nodes maintain a probability distribution over the two actions. Compared to the pure strategy game, in the mixed strategy game every node can define the strategy by which it will be active only for a fraction of the time and stay silent the rest of the time.

In the mixed strategy game, node \( i \) of class \( j \) can choose to play action \( T \) with probability \( p_{ij} \) and with probability \( 1 - p_{ij} \). Let the game profile in this multi-class framework \( p = (p_{11}, \ldots, p_{1N_1}, \ldots, p_{1j}, \ldots, p_{1N_1}, \ldots, p_{1M}, \ldots, p_{NM,M}) \).

If \( 0 < p_{ij} < 1 \), \( \forall i, j \) then \( p \) is a fully mixed strategy profile of the game. We denote by \( (p_{ij}, p_{-i}) \) the fully mixed strategy profile of the game when relay \( i \) of class \( j \) uses strategy \( p_{ij} \) and others use \( p_{-i} = (p_{11}, \ldots, p_{1j}, \ldots, p_{1N_1}, \ldots, p_{1j}, \ldots, p_{1M}, \ldots, p_{NM,M}) \).

Proposition 2: For any reward mechanism, at the mixed equilibrium, all players of the same class use the same probability: \( p_{ij} = p_{ij}, \forall i; \forall 1 \leq j \leq M \).

Proof: See the full version of the paper [5].

Let \( p_{ij} \) be the symmetric mixed strategy adopted by every node of class \( j \), \( p_{ij} = p_j, \forall i, j \). For clarity’s sake, we characterize the mixed strategy \( p_j \) in the two-class scenario without any loss of generality (\( M = 2 \)).

Proposition 3: Let \( r \) be the reward mechanism designed by sources. Then there exists a unique fully mixed Nash equilibrium \( (p_1^*, p_2^*) \) for the multi-class case. Moreover it is the solution of, \( A_1(N, p_1^*, p_2^*) = A_2(N, p_1^*, p_2^*) = 0 \) where:
\[
A_1(N, p_1^*, p_2^*) = N_1 \sum_{k_1=0}^{N_1-1} \sum_{k_2=0}^{N_2-1} (C_{k_1}^{N_1-1} p_1^{k_1} (1 - p_1^*)^{N_1-k_1}) 
\]

and
\[
A_2(N, p_1^*, p_2^*) = N_2 \sum_{k_1=0}^{N_1-1} \sum_{k_2=0}^{N_2-1} (C_{k_2}^{N_2-1} p_2^{k_2} (1 - p_2^*)^{N_2-k_2}) U_1(T, k_1 + k_2) 
\]

Moreover,
(i) if \( \frac{p_1}{p_2} = \frac{q_1}{q_2} \) then we have \( p_1 = p_2 \).
(ii) if \( \frac{p_1}{p_2} < \frac{q_1}{q_2} \) then we have \( p_1 < p_2 \). As a consequence \( k_{T,1} < k_{T,2} \).

Proof:

The utility of an active user of Class 1 is given by:
\[
U_1(p_{11}, p_{-1}) = p_1 \sum_{k_1=0}^{N_1-1} \sum_{k_2=0}^{N_2-1} (C_{k_1}^{N_1-1} p_1^{k_1} (1 - p_1)(1 - p_1) + (1 - p_1) * 0 
\]

\[
= p_1 \sum_{k_1=0}^{N_1-1} \sum_{k_2=0}^{N_2-1} (C_{k_1}^{N_1-1} p_1^{k_1} (1 - p_1) U_1(T, k_1 + k_2)) 
\]

and utility of user \( i \) from Class 2 writes
\[
U_2(p_{21}, p_{-1}) = p_2 \sum_{k_1=0}^{N_1-1} \sum_{k_2=0}^{N_2-1} (C_{k_2}^{N_2-1} p_2^{k_2} (1 - p_2) U_2(T, k_1 + k_2) 
\]

where \( A_1(N, p_1, p_2), A_2(N, p_1, p_2) \) are defined as follows:
\[
A_1(N, p_1, p_2) = \sum_{k_1=0}^{N_1-1} \sum_{k_2=0}^{N_2-1} (C_{k_1}^{N_1-1} p_1^{k_1} (1 - p_1) U_1(T, k_1 + k_2), 
\]

and
\[
A_2(N, p_1, p_2) = \sum_{k_1=0}^{N_1-1} \sum_{k_2=0}^{N_2-1} (C_{k_2}^{N_2-1} p_2^{k_2} (1 - p_2) U_2(T, k_1 + k_2), 
\]
At the Nash equilibrium we have, \( \forall \) player \( i \) of class \( j = 1, 2, \frac{\partial g_i^*(\tau)}{\partial \tau} = A_J(N, p_i^*, p_j^2) = 0 \), if \( A_J(N, p_i^1, p_j^2) < 0 \). \( p = 0 \) is the best response for the player \( i \) of class \( j \) and conversely, \( p = 1 \) is a best response when \( A_J(N, p_i^1, p_j^2) > 0 \). A mixed strategy is obtained when \( A_J(N, p_i^1, p_j^2) = 0 \), \( \forall j \in 1, 2 \). Also, we have \( A_J(N, p_i^1, p_j^2) \) is a strictly decreasing function in both \( p_1, p_2 \) (Assumption A). Thus there exists a mixed Nash Equilibrium which is unique and characterized by the equation (6).

\[
A_1(N, p_1^*, p_2^*) = A_2(N, p_1^*, p_2^*) = 0. \tag{6}
\]

Now let

\[
C(i) = \sum_{k_1=0}^{N_1-2} \sum_{k_2=0}^{N_2-2} P(k_1=k_1, k_2=k_2) r_i P_{succ}(T, k_1+k_2+e_i+1)
\]

for user \( i \), where \( e_i = 1 \) if user \( i \) is active and \( e_i = 0 \) otherwise. We can thus rewrite the expressions of \( A_1(N, p_1^1, p_2^2) \) and \( A_2(N, p_1^1, p_2^2) \) as follows:

\[
\begin{align*}
A_1(N, p_1^1, p_2^2) & = r_1 p_2 C(1) - r_1 (1-p_2) C(0) - g_1 \tau \\
A_2(N, p_1^1, p_2^2) & = r_2 p_1 C(1) - r_2 (1-p_1) C(0) - g_2 \tau 
\end{align*}
\]

It follows that, \( A_1(N, p_1^1, p_2^2) = A_2(N, p_1^1, p_2^2) = 0 \) \( \implies \)

\[
\begin{align*}
p_2 C(1) - (1-p_2) C(0) & = \frac{g_1 \tau}{r_1} \\
p_1 C(1) - (1-p_1) C(0) & = \frac{g_2 \tau}{r_2} 
\end{align*}
\]

Letting \( \frac{g_1 \tau}{r_1} = \frac{g_2 \tau}{r_2} \) we have, \( p_1 = p_2 \). This completes the proof (i).

Now, let \( \gamma_1 = \frac{g_1 \tau}{r_1}, \gamma_2 = \frac{g_2 \tau}{r_2} \) then from (9) and (10) we have:

\[
(p_2 - p_1) C(1) + (p_2 - p_1) C(0) = \gamma_1 - \gamma_2 \\
\Rightarrow (p_2 - p_1) C(0) + C(1) = \gamma_1 - \gamma_2
\]

Since, \( C(0) > C(1) > 0 \), then, \( \gamma_1 > \gamma_2 \Rightarrow p_2 > p_1 \). This tells that in order to have fewer nodes active in class 1 we should allocate smaller reward. However, if we go back to the definition of \( k_{1,1} \) and \( k_{1,2} \) in (5) we obtain \( P_{succ}(T, k_{1,1}) > P_{succ}(T, k_{1,2}) \). Under assumption A we have, \( k_{1,2} > k_{1,1} \). Hence the proof of (i).

The last result specializes the minority game to a minority game with several thresholds allowing to control the average number of active users in each class at equilibrium. Furthermore, we characterize how sources may design a reward mechanism in order to achieve the fairness of energy consumption based on the type of devices. For instance, if for the sake of fairness the objective is to incite more devices with high battery capacity (i.e., class 2) to participate in forwarding compared to small devices (i.e., class 1), our scheme under mixed equilibrium may achieve this goal by assigning a reward mechanism satisfying the relation \( r_2 > \frac{g_2 \tau}{r_2} \). In section V we will design a learning algorithm that allows sources and relays to achieve the desired performance by taking into account energy consumption; the learning algorithm converges to the full mixed equilibrium without requiring perfect state information at relay nodes.

**IV. HETEROGENEOUS USER-DEPENDENT ENERGY COST**

Energy depletion at DTN nodes depends not only on the wireless technology used by relays and on the device type but also on the device’s user behavior. Indeed, two users of identical devices may have drastically different energy-consumption rates: some "active" users will be draining their batteries much faster than other users. This suggests that, beyond physical characteristics of DTN nodes, the energy cost of DTN nodes should depend on the profile of users.

In this section we focus on how the incentive mechanism combined with the minority game framework can drive this user-dependent energy cost system to an operating point that satisfies the performance requirements in the incomplete information scenario: since the energy cost of a relay node is based on its activities and its behavior, sources cannot physically identify the energy profile of a relay node. Hence we assume that each node only knows its own specific energy cost but not those of other nodes.

On the other hand sources and relay nodes have a common information on the cumulative probability distribution function \( F(\cdot) \) of the energy costs in the system. However under incomplete information, a basic approach for sources is to consider a homogeneous reward mechanism in order to achieve their target. Homogeneity is obtained by, instead of considering the cdf \( F(\cdot) \), assuming that the sources will rely only on the mean \( \mu \) of relays energy costs distribution.

Thus, under incomplete information, the utility function for active node \( i \) becomes

\[
U_i(T) = \mathbb{E}_k [ r P_{succ}(T, k+1) - g_i \tau ] ,
\]

where the expectation is taken over \( k \) active relay nodes according to binomial distribution \( B(N-1, F(g)^\theta) \).

**Proposition 4:** For any reward mechanism \( r \), there exists a threshold-type Nash Equilibrium in which, node \( i \) is active if and only if its energy cost \( g_i \) is smaller than a threshold \( \gamma_i \leq g_{th}(r) \). Moreover the threshold \( g_{th}(r) \) is the unique solution to \( \Theta(g) = 0 \), where \( \Theta(g) := \sum_{k=0}^{N-1} C_k^{N-1} F^{k}(1-F(g))^{N-k-1} [ r P_{succ}(T, k+1) - g \tau ] \)

**Proof:** Note that a relay node \( i \) decides to be active if its utility function is positive, i.e., \( U_i(T) = \Theta(g_i) \geq 0 \). Thus it is easy to check that a relay obtains a positive utility if its energy cost \( g_i \leq g_{th}(r) \). Thus it remains to show there exists a unique solution \( g_{th}(r) \) satisfies (11).

First we will show the existence of solution \( g_{th} \) such that \( \Theta(g) = 0 \). It is easy to check that: \( \lim_{g \to 0} \Theta(g) > 0 \) and \( \lim_{g \to \infty} \Theta(g) < 0 \). Hence there exists a solution to \( \Theta(g) = 0 \). The uniqueness follows from the monotonicity of function \( \Theta \).

\( ^4 \)This comes from the fact that the more number of active nodes, the less is the probability of obtaining the reward for a tagged node.

\( ^5 \)Recall that cdf \( F(g) = P(g_i \leq g) \).
Indeed, the first term of the function $\Theta$ is decreasing function in $F(g)$ since increasing $F(g)$ engenders more active users which decreases the probability of a relay to receive a reward from sources (Assumption A). Since the function $F(g)$ is increasing in $g$, it follows that $\Theta$ is decreasing in $g$. This ends our proof of the existence of unique solution $g_{th}$ to $\Theta(g_{th}) = 0$.

We observe that reward $r^*$ satisfies the following relation

$$r^* E_{\text{succ}}(T, \Psi) = \mu r,$$  \hspace{1cm} (11)

where $\Psi$ is the number of relay nodes at Nash equilibrium. Furthermore, a relay will decide to participate if its effective energy cost is less or equal to the threshold value $g_{th}$. But the source node relies on the mean $\mu$ for the reward setting. A direct implication is that the source node may conversely provide more reward than actually needed in order to expect the targeted performance level to be achieved. This is the case when $g_{th} < \mu$. Here, expectation makes the opposite case also possible, i.e., for a certain expected value, both larger and smaller values of the cost will be present depending on relays, which may add a fluctuation dynamic of the effectively control the reward mechanism under incomplete information.

V. DISTRIBUTED REINFORCEMENT LEARNING ALGORITHM

In this section we introduce a distributed reinforcement learning algorithm that permits relays to adjust their strategies over time in the framework of the DTN MG designed in section II. The analysis of convergence of the algorithm relies on a stochastic model that is associated to a continuous time deterministic dynamics. We prove that this process converges almost surely towards an $\epsilon$-approximate Nash equilibrium.

In DTNs, nodes’ limited computational power and network’s sparsity require adaptive and distributed mechanisms letting relays adapt to operating conditions at low cost. The learning algorithm proposed here has the following attractive features:

- It is genuinely distributed: strategy updating decision is local to relays;
- It depends solely on the achieved payoffs: nodes utilize local observations to estimate their own payoffs;
- It uses simple behavioral rule in the form of logit rule.

We assume that each relay node $i$ has a vector $x_i$, which represents her perception of the payoff performance of each action (To be active, or not), and based on this perception, the tagged node takes a decision by applying a logit rule. The payoff of the chosen action is then observed and employed to update the perception for that particular action. This procedure is repeated in rounds, each of duration $\tau$, giving rise to a discrete time stochastic process which is the learning process.

For notation’s sake, denote $A = \{T, S\}$ the set of pure strategies, and $\Delta_i$ is the set of mixed strategies for relay node $i$ with $i \in \{1, ..., N\}$. Let $V^i(.)$ the payoff function for relay node $i$. The algorithm works in rounds, at round $k$, each relay node $i$ takes an action $a_i^k$ according to a fully mixed strategy $p_i^k = \sigma_i(x_i^k) \in \Delta_i$. The fully mixed strategy is generated according to the vector $x_i^k = (x_{ia}^k)_{a \in A}$ which represents its perceptions about the payoffs of the available pure strategies. In particular, relay node $i$’s fully mixed strategies are mapped from the perceptions based on the logit rule:

$$\sigma_{ia}(x_i) = \frac{e^{\beta x_{ia}}}{\sum_{a \in A} e^{\beta x_{ia}}} \hspace{1cm} (12)$$

where $\beta$ is commonly called the temperature of the logit. When $\beta \to 0$ it leads to the uniform choice of strategies, while for $\beta \to \infty$ the relay chooses the pure strategy with the largest perception. We assume that $\sigma_{ia}$ is strictly positive for all $a \in A$.

At round $k$, the perceptions $x_{ia}^k$ will determine the fully mixed strategies $p_i^k = \sigma_i(x_i^k)$ that are used by node $i$ to choose, at random, action $T$ (to be active) or $S$ (to be silent). Then each relay node estimates his own payoff $\tilde{u}_i^k$, with no information about actions or payoffs of the other relay nodes, then this value ($\tilde{u}_i^k$) is used to update relay’s perceptions as:

$$x_{ia}^{k+1} = \left\{ \begin{array}{ll} (1 - \gamma^k)x_{ia}^k + \gamma^k \tilde{u}_i^k & \text{if } a_i^k = a \\ x_{ia}^k & \text{otherwise,} \end{array} \right. \hspace{1cm} (13)$$

where $\gamma^k \in (0,1)$ are the smoothing factors that satisfy $\sum_k \gamma^k = \infty$ and $\sum_k (\gamma^k)^2 < \infty$ (an example of such factor is $\gamma^k = \frac{1}{k^2}$). A relay node only changes the perception of the strategy just used in the current round and keeps other perceptions unchanged. Algorithm (1) summarizes the learning process. The stochastic process expressed in (13) represents the evolution of relay node perceptions and can be written in the following equivalent form:

$$x_{ia}^{k+1} - x_{ia}^k = \gamma^k[u_{ia}^k - x_{ia}^k], \forall i \in \{1, ..., N\}, a \in A \hspace{1cm} (14)$$

with

$$u_{ia}^k = \left\{ \begin{array}{ll} \tilde{u}_i^k & \text{if } a_i^k = a \\ x_{ia}^k & \text{otherwise.} \end{array} \right. \hspace{1cm} (15)$$

In what follows we will prove that: although the information it needs to operate is minimal, this algorithm can attain a steady state for the coordination process among relay nodes.
A. Convergence of the Learning Process

Based on the theory of stochastic algorithms, the asymptotic behavior of (14) can be analyzed through the corresponding continuous dynamics [2]:

\[
\frac{dx}{dt} = E(w|x) - x, \tag{16}
\]

where \(x = (x_{ia}, \forall i \in \{1, \ldots, N\}, a \in A)\) and \(w = (w_{ia}, \forall i \in \{1, \ldots, N\}, a \in A)\).

Let us make equation (16) more explicit by defining the mapping from the perceptions \(x\) to the expected payoff of user \(i\) choosing action \(a\) as \(G_{ia}(x) = E(V|x, a_i = a)\).

Proposition 5: The continuous dynamics (16) may be expressed as

\[
\frac{dx_{ia}}{dt} = \sigma_{ia}(G_{ia}(x) - x_{ia}) \tag{17}
\]

Proof: Using the definition of the vector \(w\), we can compute the expected value \(E(w|x)\) by conditioning on relay \(i\)'s action:

\[
E(w_{ia}|x_{ia}) = p_{ia}U(a, p_{-i}) + (1 - p_{ia})x_{ia} = \sigma_{ia}G_{ia}(x) + (1 - \sigma_{ia})x_{ia} \tag{18}
\]

which with (16) yields (17).

This can be interpreted as follows: when the difference between the expected payoff and the perception value is large, the perception value, from (14), will be updated with a large expected value \(w_{ia} = x_{ia}\) and this difference will be reduced.

In the following theorem, we prove that the learning process admits a contraction structure with a proper choice of the temperature \(\beta\).

Lemma 1: Under the logit decision rule (12), if the temperature satisfies \(\beta < \frac{1}{n_{1+a}}\), then the mapping from the perceptions to the expected payoffs \(G(x) = [G_{ia}(x), \forall i \in \{1, \ldots, N\}, a \in A]\) is a maximum-norm contraction.

Proof: We give only a sketch of the proof in several points. For the full proof refer to [5]. Let relay \(i\) action be active (action \(T\)). Then \(G_{IT}(x) = \sum_{j=0}^{N} n_{ja}P_{\text{succ}}(T, j) C_{ja}^N(\sigma_{ia}(x_{ia}))^{(1 - \sigma_{ia}(x_{ia}))^{N-j}} - y_T\):

- We show that \(|G_{IT}(x_{ia}) - G_{IT}(\tilde{x}_{ia})| \leq n_{1+a}r|\sigma_{ia}(x_{ia}) - \tilde{\sigma}_{ia}(\tilde{x}_{ia})|\) for any two perceptions \(x_{ia}\) and \(\tilde{x}_{ia}\) of a relay node \(i\).
- Then we prove that \(\sigma_{ia}(x_{ia}) - \tilde{\sigma}_{ia}(\tilde{x}_{ia}) \leq \beta ||x - \tilde{x}||_{\infty}\).
- This concludes to \(|G_{IT}(x_{ia}) - G_{IT}(\tilde{x}_{ia})| \leq \beta n_{1+a}r||x - \tilde{x}||_{\infty}\).

Observing that since by the minority rule in\( G_{IT}(\cdot)G_{IS}(\cdot) \leq 0\), if \(\beta < \frac{1}{n_{1+a}}\), indeed \(G(x)\) is a maximum-norm contraction.

Based on the property of contraction mapping, there exists a fixed point \(x^*\) such that \(G(x^*) = x^*\). In the following theorem we show that the distributed learning algorithm also converges to the same limit point \(x^*\).

Theorem 1: If \(G(x)\) is a \(||\cdot||_{\infty}\)-contraction, its unique fixed point \(x^*\) is a global attractor for the adaptive dynamics (17), and the learning process (14) converges almost surely towards \(x^*\). Moreover the limit point \(x^*\) is globally asymptotically stable.

Proof: Since \(G(x)\) is a \(||\cdot||_{\infty}\)-contraction, it admits a unique fixed point \(x^*\). From the previously cited results in stochastic approximation together with ([7], [2] corollary 6.6), we have the almost sure convergence of (14), given that we provide a strict Lyapunov function \(\phi\).

Now let \(\phi(x) = \|x_{ia} - x^*\|_{\infty}\), then \(\phi(x^*) = 0, \phi(x) > 0, \forall x \neq x^*. \) Let \(i \in \{1, \ldots, N\}, a \in A\) be such that \(\phi(x) = |x_{ia} - x_{ia}^*|\) and \(\phi(x^*) = 0\). Since \(G_{ia}(x)\) is a maximum norm contraction, there exist a Lipschitz constant \(\xi\) such that \(G_{ia}(x) - G_{ia}(x^*) \leq \xi|x_{ia} - x_{ia}^*|\), and \(G_{ia}(x^*) = x_{ia}^*\). All together, with combined with equation (17), we can write:\(\forall x \neq x^*\)

\[
\frac{dx_{ia}}{dt} = \frac{d(x_{ia} - x_{ia}^*)}{dt} = \frac{d(x_{ia})}{dt} - \frac{d(x_{ia}^*)}{dt} \leq \sigma_{ia}(G_{ia}(x) - x_{ia}^*) - \sigma_{ia}(G_{ia}(x) - G_{ia}(x^*) + x_{ia}^* - x_{ia}) \leq \sigma_{ia}\xi|x_{ia} - x_{ia}^*| + x_{ia}^* - x_{ia} - (1 - \sigma_{ia}\xi)\phi(x) < 0,
\]

a similar argument for the case \(x_{ia} \leq x_{ia}^*\) also shows that \(\frac{dx_{ia}}{dt} < 0\), \(\forall x \neq x^*\). Thus the function \(\phi(x)\) is a strict Lyapunov function and \(x^*\) is globally asymptotically stable.

B. Approximate fully mixed Nash Equilibrium

From lemma (1) and theorem (1), we have:

\[
G_{ia}(x^*) = E(V|x^*, a_i = a) = x_{ia}^*.
\]

This is a property of the equilibrium \((x^*)\) of the distributed learning algorithm: its value \(x_{ia}^*\) is an accurate estimation of the expected payoff in the equilibrium. Moreover we show that the fully mixed strategy

\[
p^* = (\sigma_{ia}^* = \frac{e^{\beta E_{ia}(x^*)}}{e^{\beta E_{ia}(x^*)} + e^{\beta E_{ia}(x^*)}}, \forall a \in A, i \in \{1, \ldots, N\})
\]

is an approximate Nash equilibrium.

Proposition 6: Under the Logit decision rule (12), the fully mixed strategy \(p^* = \sigma^*(x^*)\) at the equilibrium \(x^*\) is a \(\epsilon\)-approximate Nash equilibrium for our game with \(\epsilon = -\frac{1}{\beta} \sum_{a \in A} \sigma_{ia}^*(ln(\sigma_{ia}) - 1)\).

Proof: A well-known characterization of the logit gives:

\[
\sigma_{ia}(x^*) = \arg \max_{a \in A} \sigma_{ia} E(V^1|x^*, a_i = a) - \frac{1}{\beta} \sum_{a \in A} \sigma_{ia}(ln(\sigma_{ia}) - 1) = \frac{e^{\beta E_{ia}(x^*)}}{e^{\beta E_{ia}(x^*)} + e^{\beta E_{ia}(x^*)}} = e^{\beta E_{ia}(x^*)}
\]

and since ([3], pp.93) \(\max_{a \in A} \sum_{a \in A} \sigma_{ia} E(V^1|x^*, a_i = a) - \frac{1}{\beta} \sum_{a \in A} \sigma_{ia}(ln(\sigma_{ia}) - 1) \leq \max_{a \in A} \sum_{a \in A} \sigma_{ia} E(V^1|x^*, a_i = a)\) then, we have:

\[
\sum_{a \in A} \sigma_{ia} E(V^1|x^*, a_i = a) \geq \max_{a \in A} \sigma_{ia} E(V^1|x^*, a_i = a) - \epsilon
\]

where \(\epsilon = \max_{i \in \{1, \ldots, N\}} -\frac{1}{\beta} \sum_{a \in A} \sigma_{ia}(ln(\sigma_{ia}) - 1))\).

Hence the fully mixed strategy \(p^* = \sigma^*(x^*)\) in the equilibrium \(x^*\) is a \(\epsilon\)-approximate Nash equilibrium.
between nodes follows an exponential distribution with parameter \( \lambda \). Furthermore, we assume that upon successful delivery of a message, the relay node receives a positive reward \( R \) if and only if it is the first one to deliver the message to the corresponding destination. Under these assumptions, we can obtain the expressions of different quantities: in particular the probability that an active node relays a copy of a received packet to destination within time \( \tau \) is \( 1 - Q_\tau \) where the expression of \( Q_\tau \) is given by: \( Q_\tau = (1 + \lambda \tau) e^{-\lambda \tau} \). Now, the probability of successful delivery of the message for an active node is:

\[
P_{\text{succ}}(T, k_T) = \frac{1 - Q_T^{k_T}}{k_T} \tag{19}
\]

such that each node seeks to be the first to deliver a given message to its destination (see [5]).

We examine the performance of our algorithm in a multi-class framework (heterogeneous DTN), where we consider the existence of two classes of nodes. The parameters \( \lambda = 0.03 \), \( \tau = 100 \) are used throughout the numerical analysis.

The performance of the learning algorithm in the heterogeneous DTN is investigated in two cases, symmetric (i.e. when \( g_1 = \frac{g_2}{r_2} \)) and asymmetric (\( g_1 \neq \frac{g_2}{r_2} \)). We consider first the symmetric case. We consider \( g_1 = 0.8 \times 10^{-4}, g_2 = 0.5 \times 10^{-4}, r_2 = 0.15 \). Setting \( r_2 = 0.15 \) we obtain \( r_1 = 0.24 \). In Fig. (1)(a) we observe that the probability of being active of nodes of both classes \( (p_1, p_2) \) converges to the symmetric Nash equilibrium discussed in proposition (3), and the value it converges to \( (p_1^*, p_2^*) = 0.78 \) is the solution of the equation \( (A_1(N, p_1^*, p_2^*) = A_2(N, p_1^*, p_2^*) = 0) \). The average number of active nodes, depicted in Fig (1)(b), converges to \( \Psi = 30 \) that satisfies the relation (3).

In Fig(2), we depict the asymmetric case, when \( g_1 > g_2 \) and \( r_1 < \frac{g_2}{r_2} \). In Fig(2)(a) we observe that \( (p_2 > p_1) \), in other words, the nodes with high energy constraint (class 1) are less active, thus by allocating smaller reward \( (r_1) \), fewer nodes of class 1 are active. Notice in Fig(2)(b) that the average number of active nodes \( \Psi_1 \leq k_T < \Psi_2 \).

VI. APPLICATION AND NUMERICAL RESULTS

In this section, we provide a numerical analysis of the performance achieved by DTN nodes following the distributed reinforcement learning mechanism proposed in section V.

For the rest of the paper, we will assume that relay nodes use the two hop routing scheme, in which a relay node receives a copy of the message from the source and can only forward it to the destination node. We assume the inter-meeting rate between nodes follows an exponential distribution with parameter \( \lambda \). Furthermore, we assume that upon successful delivery of a message, the relay node receives a positive reward \( R \) if and only if it is the first one to deliver the message to the corresponding destination. Under these assumptions, we can obtain the expressions of different quantities: in particular the probability that an active node relays a copy of a received packet to destination within time \( \tau \) is \( 1 - Q_\tau \) where the expression of \( Q_\tau \) is given by: \( Q_\tau = (1 + \lambda \tau) e^{-\lambda \tau} \). Now, the probability of successful delivery of the message for an active node is:

\[
P_{\text{succ}}(T, k_T) = \frac{1 - Q_T^{k_T}}{k_T} \tag{19}
\]

such that each node seeks to be the first to deliver a given message to its destination (see [5]).

We examine the performance of our algorithm in a multi-class framework (heterogeneous DTN), where we consider the existence of two classes of nodes. The parameters \( \lambda = 0.03, \tau = 100 \) are used throughout the numerical analysis.

The performance of the learning algorithm in the heterogeneous DTN is investigated in two cases, symmetric (i.e. when \( g_1 = \frac{g_2}{r_2} \)) and asymmetric (\( g_1 \neq \frac{g_2}{r_2} \)). We consider first the symmetric case. We consider \( g_1 = 0.8 \times 10^{-4}, g_2 = 0.5 \times 10^{-4}, r_2 = 0.15 \). Setting \( r_2 = 0.15 \) we obtain \( r_1 = 0.24 \). In Fig. (1)(a) we observe that the probability of being active of nodes of both classes \( (p_1, p_2) \) converges to the symmetric Nash equilibrium discussed in proposition (3), and the value it converges to \( (p_1^* = p_2^* = 0.78) \) is the solution of the equation \( (A_1(N, p_1^*, p_2^*) = A_2(N, p_1^*, p_2^*) = 0) \). The average number of active nodes, depicted in Fig (1)(b), converges to \( \Psi = 30 \) that satisfies the relation (3).

In Fig(2), we depict the asymmetric case, when \( g_1 > g_2 \) and \( r_1 < \frac{g_2}{r_2} \). In Fig(2)(a) we observe that \( (p_2 > p_1) \), in other words, the nodes with high energy constraint (class 1) are less active, thus by allocating smaller reward \( (r_1) \), fewer nodes of class 1 are active. Notice in Fig(2)(b) that the average number of active nodes \( \Psi_1 \leq k_T < \Psi_2 \).

VII. DISCUSSION OF ASSUMPTIONS, LIMITATIONS AND FUTURE WORK

In this section we discuss the main assumptions that were adopted to yield a tractable model and we describe limitations and possible extensions.

**Mobility pattern**: A key challenge in developing our results has been to make general assumptions about the mobility of DTN nodes. In particular, the properties derived for our incentive mechanism hold under any homogeneous mobility pattern. Indeed, the large majority of analytical studies are based on some assumptions on the mobility for the sake of tractability. Early works typically assumed that the cdf of inter contact time decays exponentially over time such as in random waypoint models. However, extensive empirical mobility traces later show that the CCDF of the inter-contact time follows approximately a power law over large time range with exponent less than unit [6].

A further assumption to prove convergence of our stochastic approximation algorithm is that nodes are identical and uniformly visit the entire network space. Experimental data, however, have shown that mobility patterns of individuals are typically restricted to a given area, and the overall node density is often largely inhomogeneous. Such models allow studying how DTN routing mechanisms are affected by highly inhomogeneous node density, diverse mobility patterns and transmission technologies. In future work, we will adapt our mechanism for cases where heterogeneity is caused under human mobility during a day. We will study how to model such changes and how routing algorithms can take into account these time-of-the-day effects.

**Buffer management**: In our model, we assume that relay nodes have enough capacity to store messages generated from sources and their copies. But it is clear that, in the context of DTNs, node buffers may well overflow if no message discarding policy is adopted. In turn, performance measures for DTNs depend not only on the number of active relay nodes but also on buffer capacity. In this scenario, efficient drop policies at relay nodes decide which messages should prioritized under capacity constraints regardless of the specific routing algorithm used.

**Delivery probability**: A central performance measure studied in DTNs literature is delivery probability. This measure holds relevant for several applications which may be running over DTNs. However our scheme can be designed to attain general performance metrics provided that they satisfy assumption A;
for instance, end-to-end delay is such a metric.

**Routing and Protocols:** We did not address specific protocols for the delivery of messages, rather we have considered an incentive scheme where only successful relays receive a reward. In turn, this scheme is indeed general and the proposed framework in principle can work for any DTN routing protocol. But, the assumption about how a relay node obtains a reward may limit coordination between relay nodes. In fact, in order to avoid the use of feedbacks that allow relays to know whether the message has been successfully delivered or not, we assume that a relay will receive a reward if and only if it is the first one to deliver the message. Unfortunately, this reward scheme may foster unintended deviant behaviors in which relay nodes may refuse to forward messages to other relays in order to increase their own utility [1]. In the future, we will propose a modification of our mechanism that can eliminate this problem by using some additional rewards between relays in order to incentivize messages propagation between them [1].

**VIII. Conclusion**

In this paper we have devised a rewarding scheme where relays gain certain rewards that incentive them to sacrifice memory and battery on DTNs relaying operations. Furthermore, we make a specific effort such in a way that our mechanism is designed to account for heterogeneous user profiles and devices energy costs.

Furthermore we argue that any such a coordination scheme should not rely on end to end control message exchange. To this respect, our paper provides a novel key contribution: the reward mechanism in fact is designed using the theory of Minority Games (MGs) in order to attain coordination in distributed fashion. Overall, our scheme covers several possible information scenarios that sources and relay nodes may face in reality, ranging from full state information to imperfect state information and applies to general intermeeting distributions for nodes’ contacts.

Also, in order to prove the correctness of our incentive mechanism, we have provided a complete characterization of the equilibria of the baseline MG in the case of heterogeneous DTNs. Finally, the core machinery to attain feedback-less coordination is based on a learning algorithm that involves stochastic approximations: our algorithm provably drives the system to the aforementioned equilibria while requiring just local estimations of system parameters performed at mobile relay nodes.

**References**


