Beyond Best Effort: Choosing Connectivity Portfolio for Cloud Brokering Platform by Risk/Profit Tradeoff

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***Abstract*—We utilize portfolio theory of finance and investment science in order to show how a cloud broker can select a portfolio of connectivities in order to accommodate requirements for cloud services of users, who need better than the best effort Quality of Service (QoS). Following economic theory, we assume that cloud broker makes his decision by choosing appropriate tradeoff between profit and risk. The risk has two sources: uncertain demand and uncertain connectivity quality. Using Internet traffic data we show that it is possible to deliver considerably higher QoS by selection of connectivity portfolios that combine the best effort connectivity of different grades with premium grade connectivity.**

***Keywords*-cloud broker, best effort traffic, carrier grade connectivity, portfolio of connectivities, risk/return tradeoff**

I. Introduction

Current research literature recognizes that cloud broker gradually emerges as the central actor in cloud ecosystems, see [1], [2]. This actor provides a platform that facilitates connection between heterogeneous user population and equally heterogeneous service offers from multiple cloud service providers. In particular, the cloud broker brings the added value to the users of cloud services by providing intermediation, aggregation and arbitrage.

Different technical and organizational issues of cloud brokering were considered in [3], [4] among others. However, economic analysis of cloud brokering was not given sufficient attention in this literature, especially from the point of view of uncertainties and risks present both from demand and supply sides of cloud ecosystem. This paper aims at filling this gap. More specifically, we develop a set of models that allow to shed light on the following issues

- The economic and financial effects of brokering SaaS offerings and its bundling with carrier grade connectivity and other connectivity options.

- How cloud broker can choose the optimal and/or acceptable trade-off between risks and profits in cloud ecosystem.

The novelty of this paper consists in extending the modern portfolio theory of investment science [5] to the case of complex brokering platforms that connect populations of users with providers of modular cloud services. This analysis involves the definition of relevant activities and risks. In particular, we focus on the quality of service and the risk is associated with providing the service of quality inferior to user's expectations and service level agreements that can lead to users dissatisfaction and erosion of users base. Then the portfolio of activities consists of selection of Quality of Service (QoS) enhancing measures. Here we concentrate on one such measure, which is the selection of connectivity providers that provide connectivity of different grades, like best effort connectivity or carrier grade connectivity. Other measures like scheduling of tasks and their synchronization between them are beyond the scope of this paper, even though the portfolio approach can be applied there too. To the best of our knowledge, the portfolio analysis of business models in information and telecommunication industry from the point of view of balance between performance and relevant risks has not received attention in the research literature, [6] being an exception.

We present concise background on cloud brokering in Section II that serves as the motivation for the subsequent modeling. In Section III we focus on risk, generated by users dissatisfaction with QoS that the cloud broker controls by selecting a portfolio of connectivity options. For the sake of clarity we concentrate on portfolios with just two components: carrier grade connectivity and the best effort connectivity. The general problem with arbitrary number of connectivity options is considered in Section IV, where we provide the analysis of structure of resulting efficient connectivity portfolios. A sample of extensive numerical experiments with several connectivity options using real Internet traffic data is presented in Section V. Its subsection V-A summarizes the business insights obtained from these experiments.

II. Cloud Brokering Environment

In this paper we consider the offer of Cloud Broker to Small and Medium Enterprizes (SME) market. He acts as single point of contact (SPOC) for the SME's and offers a cloud service portfolio with a hassle free implementation and usage of business applications with no IT or infrastructure cost, a pay-as-you-go model and a single Service-Level Agreement (SLA). This is regarded as a strong value proposition and a major differentiator. According to [7], enterprises will pay for service portfolios with premium cloud enhancements, such as low latency and bandwidth, compared with existing public cloud services. Moreover they found that carrier-grade public cloud services have ten times the revenue potential of basic cloud services. This serves as a motivation for our study of economic effects of enhancing the best effort connectivity in the context of cloud brokering. Reducing the investment and upgrade costs of the carriers network through an efficient operation of the connectivity resources is an additional motivation for the study.

Current XaaS providers may offer two quality levels of network transport of their cloud services: 1) OTT (Over The Top) service and 2) Assured Quality Service (AQS). The former level implies a network agnostic approach, where typically best

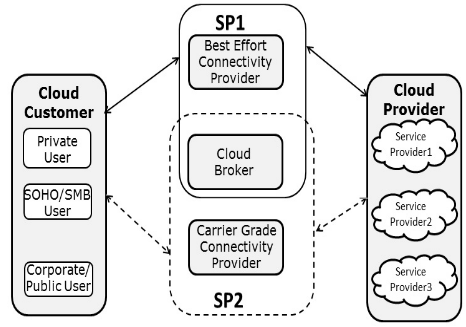

Figure 1. Supplementing best effort connectivity for SaaS brokering with carrier grade connectivity

effort public Internet is applied. The latter level is provided currently only through on-net solutions, i.e. where the provider and the user are connected to the same managed network. High quality AQS will be referred to as carrier grade connectivity in this paper.

The combination of cloud brokering with better than best effort connectivity can be viewed in the context of current Internet economics as a possible business strategy of Internet Service Providers (ISP) aimed at larger participation in cash flows generated by Internet content provision. It can be viewed as a form of vertical integration, see [8] for discussion how it is connected with the current discussion on Internet neutrality.

III. Bundling of brokering SaaS offerings with carrier grade connectivity

In order to be specific, let us consider a simplified but meaningful case shown on Figure 1. This figure shows major actors with two service package offerings in a cloud computing eco system.

The Cloud Broker (CB) provides two service portfolios SP1 and SP2 for population of homogeneous users. Service pack SP1 consists of collection of brokered SaaS services obtained from one or more SaaS providers. This service pack is delivered by Cloud Broker in association with a best effort Internet provider (CP). Service pack SP2 is delivered by Cloud Broker in association with a cloud carrier providing premium quality connectivity. It can substantially increase the Quality of Service (QoS) for critical applications. Analyzing this situation, we are interested in the answers to the following questions

- How Cloud Broker should decide which of these service packs to offer and in what proportion, based on user's expectations of QoS and related costs.

- What is the optimal trade-off between profits, costs and risks of not honoring the service level agreements with demanding users and, consequently, facing increased churn.

In order to answer these questions we have to describe quantitatively the user's demand, QoS and how the carrier grade connectivity is different from the best effort from QoS viewpoint.

Thus, the *users* generate demand d for brokered SaaS services, which is described by a random variable with cumulative distribution function (cdf) $H(y)$. This demand require a specific QoS. The portion of demand that is satisfied with lesser quality is considered to be not satisfied and lost. In case of unsatisfied demand CB is penalized with the unit penalty c which is composed both from direct revenue loss and indirect revenue loss due to potential churn. The unit of satisfied demand brings to CB the revenue p. Suppose that CB can satisfy amount of demand v with required quality of service. Then the profit resulting from demand satisfaction is

$$p\mathbb{E}\min\{d,v\} - c\mathbb{E}\max\{0,d-v\} \qquad (1)$$

For the reason of less than perfect QoS the satisfied demand v is itself a random variable, such that the expectation in (1) is taken both with respect to d and v. In this paper we concentrate on the component of QoS that is defined by the quality of connectivity.

In the situation shown on Figure 1 CB has two options for the connectivity selection. Firstly, he can get decided amount of the best effort connectivity u from Connectivity Provider. The cost of this decision for CB is $c_{0u} + c_{1u}u$ where c_{0u} is the fixed cost and c_{1u} is the variable cost. The connectivity is measured here in the same units as demand and ideally the connectivity u will satisfy the amount of demand u. However, in order to describe the *best effort* grade of this connectivity it is assumed that only a random portion of demand u will be satisfied with required QoS. Specifically, the amount of demand that is satisfied by connectivity u equals $(1-z_u)\,u$, where $z_u \geq 0$ is a random variable that takes values between 0 and 1 with cdf $G_u(y)$.

Alternatively, CB can develop the carrier grade connectivity of size x for the cost $c_{0x}+c_{1x}x$. Similarly to the previous case, the amount of demand it satisfies equals $(1-z_x)\,x$, where $z_x \geq 0$ is again a random variable that takes values between 0 and 1 with cdf $G_x(y)$.

Generally, the policy of CB may consist in the mixture of the two policies described above: get some amount of connectivity u from CP and establish amount x of carrier grade connectivity. Then the total satisfied demand v from (1) is

$$v = (1-z_u)\,u + (1-z_x)\,x$$

Substituting this into (1) and taking into account connectivity provision costs we obtain the profit

$$\begin{aligned} P(x,u) = p\mathbb{E}\min\{d,(1-z_u)\,u+(1-z_x)\,x\} - \\ c\mathbb{E}\max\{0,d-(1-z_u)\,u-(1-z_x)\,x\} - \\ c_{0u}y_u - c_{1u}u - c_{0x}y_x - c_{1x}x \qquad (2) \end{aligned}$$

where $y_u = 1$ if $u > 0$ and zero otherwise; where $y_x = 1$ if $x > 0$ and zero otherwise.

By maximizing profit (2) with respect to x and u it is possible to obtain the optimal mixture (or portfolio) of two types of connectivity. However, such strategy will not take into account different risks that this decision will carry, which can be substantial. For this reason the connectivity portfolio should not only maximize the expected profit, but also help to control the risks. In order to model this trade-off it is necessary to define the notion of risk that is relevant in the context of cloud brokering.

In this section we shall follow the industrial practice and assume that the main risk to Cloud Broker comes from possible violation of service level agreements, user's expectations about QoS and resulting potential increase in *churn* (user

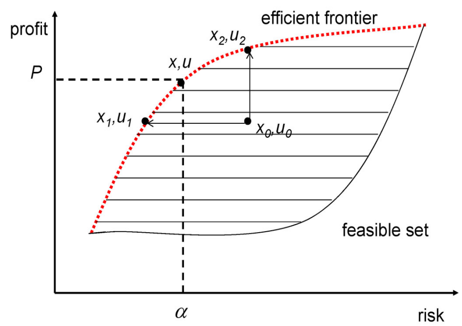

Figure 2. Efficient connectivity portfolios

abandonment of service in favor of competition). Such risk can be measured by the share of demand that is not satisfied with required QoS:

$$\frac{1}{\bar{d}}\mathbb{E}\max\left\{0, d - (1 - z_u)\, u - (1 - z_x)\, x\right\} \quad (3)$$

where $\bar{d} = \mathbb{E}d$ is expected demand. Alternatively to (3), one can measure risk by some degree of profit variability like variance [5] or Value-at-Risk [9] [10].

Taking into account this risk, the connectivity portfolio (x, u) will be selected by maximizing profit (2) under condition that the fraction of lost demand (3) will not exceed some threshold α :

$$\max_{x,u\geq 0} P(x, y) \quad (4)$$

$$\mathbb{E}\max\left\{0, d - (1 - z_u)\, u - (1 - z_x)\, x\right\} \leq \alpha\bar{d} \quad (5)$$

Solution of (4)-(5) gives the optimal trade-off between profit and risk and yields the efficient connectivity portfolio to Cloud Broker. Solving this problem for different values of risk α one obtains *efficient frontier* in the space of profit/risk [5] similar to one on Figure 2. To each point on efficient frontier corresponds *efficient portfolio*. They dominate all other possible connectivity portfolios.

Now the Cloud Broker can tune his connectivity portfolio to requirements of particular user groups by selecting the risk threshold α according to their preferences and choosing the connectivity portfolio (x, u) from corresponding point on the efficient frontier as shown on Figure 2. In section V we provide computations of efficient frontier, which have shapes similar to Figure 2.

IV. Selection of connectivity portfolio among several options

In this section we consider a more realistic case when the Cloud Broker can choose between arbitrary number of connectivity options $i \in \mathcal{N} = \{1, ..., n\}$ connectivity options. His decision takes the form of connectivity portfolio $x = (x_1, ..., x_n)$, where $x_i \geq 0$ is the amount of connectivity of each type to utilize for satisfaction of users' demand.

As before, we assume that only fraction $1 - z_i$ of this connectivity satisfies the demand with required QoS, where z_i is a random variable distributed on $[0, 1]$. Therefore the maximal amount of demand satisfied with required QoS is $\sum_{i=1}^{n}(1 - z_i)\, x_i$.

By c_i and $\hat{c}_i$ we shall denote variable unit costs and fixed costs required to obtain amount of connectivity x_i. Denoting $y_i = 1$ if $x_i > 0$ and zero otherwise, similarly to (2) we obtain the following expression for CB's profit:

$$P(x, y) = p\mathbb{E}\min\left\{d, \sum_{i=1}^{n}(1 - z_i)\, x_i\right\} - \quad (6)$$
$$c\mathbb{E}\max\left\{0, d - \sum_{i=1}^{n}(1 - z_i)\, x_i\right\} -$$
$$\sum_{i=1}^{n} c_i x_i - \sum_{i=1}^{n} \hat{c}_i y_i$$

Similarly to Section III we shall measure risk by fraction of demand not satisfied with required QoS. Assuming that this fraction should not exceed α, we obtain the following constraint that admissible connectivity portfolios must satisfy:

$$R(x) \leq \alpha\bar{d},\ \ R(x) = \mathbb{E}r(x, d, z), \quad (7)$$
$$r(x, d, z) = \max\left\{0, d - \sum_{i=1}^{n}(1 - z_i)\, x_i\right\}$$

where $\bar{d}$ is the average demand. Similarly to (4)-(5) the efficient connectivity portfolio that provides the required balance between risk and profit will be found by maximizing the profit (6) subject to constraint (7). The efficient portfolio for zero fixed costs is obtained by solving a system of equations:

Proposition 1. Let H be the cdf of demand d. For any $I \subseteq \mathcal{N}$ lets denote $x^I(\mu) = \left(x_1^I(\mu), ..., x_n^I(\mu)\right)$ a solution of equations

$$\mathbb{E}_z\left[(1-z_i) H\left(\sum_{k=1}^{n}(1-z_k) x_k\right)\right] = \tag{8}$$

$$1-\bar{z}_i-\frac{c_i}{c+p+\mu},\ i \in I;\ x_i=0,\ i \in \mathcal{N} \setminus I$$

with all $x_i^I(\mu)$, $i \in I$ positive, if such solution does not exist we take $x_i^I(\mu) = 0, i \in \mathcal{N}$. Suppose that $x(\mu) = x^{\hat{I}}(\mu)$ is such that

$$P\left(x^{\hat{I}}(\mu), 0\right) = \max_{I \subseteq \mathcal{N}}\left[P\left(x^I(\mu), 0\right) - \mu R(x^I(\mu))\right] \tag{9}$$

and μ^* is the solution of equation

$$R(x(\mu)) = \alpha \bar{d} \tag{10}$$

Then portfolio x^* that maximizes (6) subject to (7) with $\hat{c}_i = 0$ exists and

$$x^* = \begin{cases} x(0) & \text{if} \quad R(x(0)) \leq \alpha \bar{d} \\ x(\mu^*) & \text{otherwise} \end{cases} \tag{11}$$

The representation (8),(10) allows to find numerically the efficient connectivity portfolios.

V. Computations of Efficient Connectivity Portfolios

We have conducted extensive set of numerical experiments with portfolio model of Section IV, deriving the distributions of connectivity quality parameters z_i from packet delay data between different test sites of RIPE Network Coordination Centre [11]. In the subsequent discussion we concentrate on the randomness in connectivity quality, because the analyzed decisions consist of selection of connectivities. The randomness of demand is also important, but it should be analyzed in detail in the context of nonhomogeneous user population, which is beyond the scope of this paper.

A. Summary of findings and business insights

1. Truncated normal distribution (TND) of connectivity quality parameters z_i can be a reasonable first approximation to their distribution based on measurements of Internet traffic. Sometimes the quality of TND approximation is excellent. In other cases the traffic measurements yield distributions that are far from TND. However, the *qualitative* patterns of resulting efficient frontiers and efficient connectivity portfolios provide an adequate insight into their properties also with real data.

2. For many practically important problems the solution of portfolio problem (6),(7) can be obtained by standard commercial software like Matlab, which combines numerical integration with nonlinear programming on laptop grade hardware.

3. It is important to have more than two connectivity alternatives. Addition of the third alternative substantially improves the profit/risk balance by increasing the profit for the same risk. Addition of the fourth alternative also improves the profit/risk balance, but substantially less than addition of the third alternative.

4. When the connection options provide relatively poor quality of service individually, their combination improves QoS substantially and allows to reduce the usage of carrier grade option.

5. Still, for very high QoS there is no substitute for carrier grade connectivity, even if its range of utilization can be reduced by selection of several independent best effort alternatives.

6. The best connectivity portfolio can not be found by using just ad hoc empirical rules and depends on non obvious interplay between costs and quality. One should solve the problem (6),(7) in order to obtain efficient connectivity portfolios. The approach of portfolio theory allows to obtain the connectivity portfolio that implements the best risk/return tradeoff within given risk tolerance.

B. Distributions of connectivity quality z_i derived from Internet traffic data

Measurement and statistics of Internet traffic and its influence on QoS of different services is a subject of intensive empirical and theoretical studies, see for example [12], [13], [14] among others. This research provides the evidence of rich variety of traffic patterns and takes nonparametric approach for its analysis.

Substantial research is dedicated to identifying relationships between the QoS for different services and the traffic characteristics. The main such characteristics are packet delay, loss and delay jitter (variance of delay), which are important for QoS of real time services [13], [14]. We have taken the nonparametric approach in the spirit of [13] to derive the distributions of connectivity quality z_i from Internet traffic data, proceeding as follows.

1. *Selection of traffic data*. We have downloaded one month of packet delay data between different test sites of RIPE Network Coordination Centre [15]. Each data set contained 60000-80000 packet delay observations in consecutive time periods.

2. *Selection of traffic characteristic*. We have selected jitter as the traffic characteristic that influences the QoS, see [13], [14] for discussion of this choice. We have made this selection because jitter defines to large extent the QoS of real time services that can be an important component of cloud broker service pack. The jitter data is computed straightforwardly from the packet delay data obtained from RIPE NCC. This yielded N_1 =60000-80000 observations of jitter in consecutive periods of time between different test sites.

3. *Extraction of distribution of connectivity quality z_i from traffic data*. This is done through the following steps for each selected pair i of test sites.

i. The whole horizon of N_1 jitter observations was divided into equal intervals of length N_2.We have assumed that the required QoS is not satisfied if jitter exceeds a given threshold a. For each interval $k = 1 : N_3$, $N_3 = \lfloor N_1/N_2 \rfloor$ we have computed the fraction z_i^k of jitter observations that exceed this threshold. This fraction takes values k/N_2 for $k = 0 : N_2$. In this way we have obtained N_3 observations z_i^k of connectivity quality z_i between selected pair i of test sites. Such subsequent intervals for estimation of traffic characteristics were considered, for example, in [16].

ii. Nonparametric density function of distribution of z_i was obtained from observations z_i^k by using kernel smoothing density estimation with normal kernel function [17].

All obtained in this way distributions of z_i fall into three types, see Figure 3 for example.

1. *Truncated normal distribution (TND)*. In can provide an excellent description of connectivity quality. An example describing connectivity between RIPE test sites 181 and 105 is shown as connectivity 4 on Figure 3.

2. *Mixture of TND and a heavy tail distribution*. An example describing connectivity between RIPE test sites 147 and 105 is shown as connectivity 2 on Figure 3.

3. *Multimodal distribution*. In this case the distribution of z_i has two modes: one is high quality mode near $z_i = 0$ and another is low quality mode around z_i =0.2-0.6. Similar multimodality was observed in [13]. An example describing connectivity between RIPE test sites 7 and 105 is shown as connectivity 3 on Figure 3.

C. Some numerical examples

We present experiments with four connectivity options. The option 1 is the carrier grade connectivity with $z_i \equiv 0$, that is the absolute quality. We have three best effort options 2,3,4 described above, see Table 1. The cloud broker is in possession of option 1 connectivity in his role as network operator with low fixed costs and high variable costs. Option 2 can also be interpreted as the carrier grade connectivity of lesser than absolute quality.

The variable cost c_1 of option 1 is normalized to 1, all other costs are taken relative to this cost. Besides, we took $p = 1.17$, $c = 1.33$. The demand was distributed according to TND truncated on $[0, \inf)$ from normal with mean $\bar{d} = 10$ and standard deviation $\sigma = 0.25$.

type	1	2	3	4
mean	0	0.089	0.323	0.444
st. deviation	0	0.103	0.160	0.117
variable costs	1	0.67	0.52	0.42
fixed costs	0.12	0.33	0.17	0.17

Table 1. *Parameters of connectivities z_i*

Figures 4-7 show efficient frontiers and corresponding efficient portfolios, illustrating different conclusions from Section V-A. Figure 4 compares efficient frontiers when two inferior quality best effort connectivities 3 and 4 are added to the basic selection of carrier grade connectivity 1 and a reasonable quality best effort type 2. One can see that in the range between low and moderate risk adding cheaper and lesser quality options may

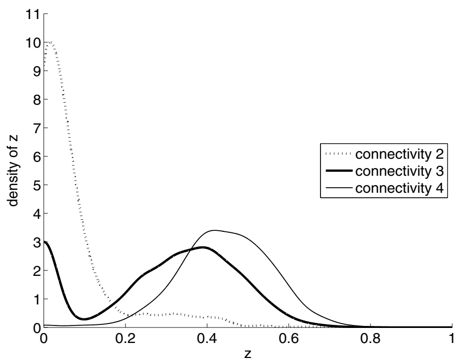

Figure 3. Probability density of z_i

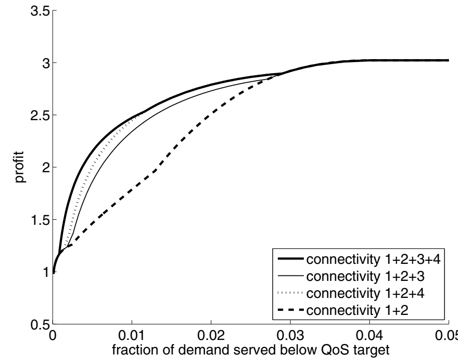

Figure 4. Efficient frontiers for different combinations of connectivities, example 1

lead to a significant improvement of profit for the same risk. The main profit increase occurs with adding of just one option. Adding the second option also improves profit, but to a lesser degree. For example, adding type 4 to types 1,2 improves profit for up to 40%, while further addition of type 3 further improves profit for up to 24%.

Figure 5 shows how the composition of efficient portfolio changes with relaxation of bounds on admissible risk. Only carrier grade connectivity is used for the highest QoS. Lower, but still high QoS is served by combination of all best effort connectivities. In the medium risk range the efficient portfolio is comprised from the best and most expensive option 2 and the worst and the cheapest option 4. Finally, in the high risk tolerance portfolio only the highest and most expensive best effort option remains. However, the comparison with portfolio that comprises only option 4 in this case (see curve "connectivity 1+4" on Figure 7) shows that the utilization of higher grade option 2 leads to significantly larger profit in the high risk range. Thus, the composition of efficient portfolio depends on intricate relationship between quality and cost which can be identified only by solving the problem (6),(7) and not by ad hoc rules.

Figures 6, 7 illustrate two additional aspects mentioned in Section V-A. Figure 6 shows what happens if the high quality carrier grade connectivity is excluded from connectivity options. On this figure the thin dashed lines show how the

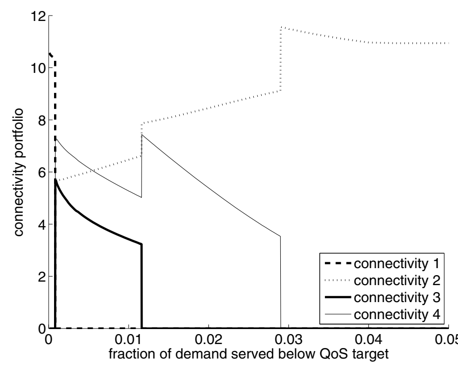

Figure 5. Efficient connectivity portfolios for combination of all four connectivity types

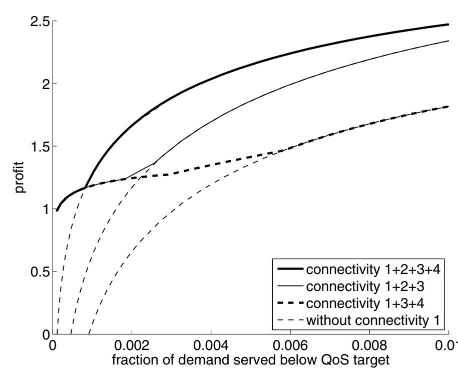

Figure 6. Efficient frontiers for different combinations of connectivities, example 2

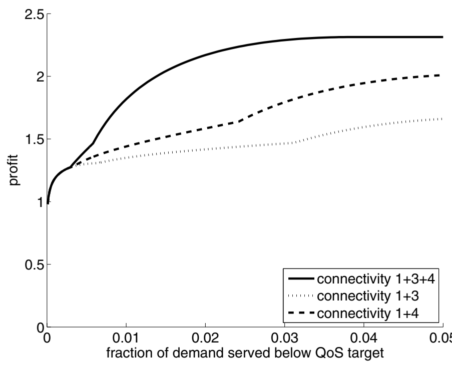

Figure 7. Efficient frontiers for different combinations of connectivities, example 3

dependence between profit and risk changes with such exclusion. One can see that in the range of medium and high risk nothing changes because the expensive premium quality connectivity does not enter efficient connectivity portfolio in these ranges. However, in the low risk high QoS range the profit drops drastically confirming that carrier grade connectivity is indispensable for economic provision of high quality service.

Finally, Figure 7 shows that combination of two low quality best effort options (3 and 4) with high quality option 1 may give much better profit in the medium and high risk zone, compared with combination of just one low quality best effort option (either 3 or 4) with the carrier grade option 1. One can see that this improvement can reach as high as 40%.

VI. Summary

In this paper we have adopted and extended portfolio theory of modern investment science in order to show how a Cloud Broker can select a portfolio of connectivity options for his service packs in order to strike the optimal balance between profit and appropriately defined risk, facing demand from user population with specific Quality of Service. The approach remains valid for nonhomogeneous user population and more complex price structures.

VII. Appendix

Proof of Proposition 1. The objective function $P(x,0)$ in (6) is concave with respect to x and

$$P(x,0) \geq p\bar{d} - \sum_{i=1}^{n} c_i x_i \to -\infty \text{ as } x \geq 0, \ \ \|x\| \to \infty.$$

Therefore the solution of this problem with or without constraint (7) exists and any its local maximum will be the global one.

1. Let us consider unconstrained maximization of (6) with zero fixed costs $\hat{c}_i$. It is equivalent to

$$\max_{x_i \geq 0} F(x), \ \ F(x) = -(c+p) \times \quad (12)$$

$$\mathbb{E} \max \left\{ 0, d - \sum_{i=1}^{n} (1-z_i) x_i \right\} - \sum_{i=1}^{n} c_i x_i$$

Let $\hat{x}$ be its solution. The Karush-Kuhn-Tucker (KKT) optimality conditions [18] state:

$$\frac{\partial}{\partial x_i} F(\hat{x}) \leq 0, \ \ i \in \mathcal{N} \text{ and } \hat{x}_i = 0 \text{ if } \frac{\partial}{\partial x_i} F(\hat{x}) < 0, \quad (13)$$

Taking into account that

$$\frac{\partial}{\partial x_i} \mathbb{E}_d \max \left\{ 0, d - \sum_{k=1}^{n} (1-z_k) x_k \right\} = \quad (14)$$

$$-(1-z_i) \left(1 - H \left(\sum_{k=1}^{n} (1-z_k) x_k \right) \right)$$

we obtain from (12):

$$\frac{\partial}{\partial x_i} F(x) = (c+p) \, \mathbb{E}_z \left[(1-z_i) \times \right.$$

$$\left. \left(1 - H \left(\sum_{k=1}^{n} (1-z_k) x_k \right) \right) \right] - c_i.$$

From this and (13) follows that $\hat{x} = x(0)$ from (9) is the solution of (12) and if in addition also $R(\hat{x}) \leq \alpha\bar{d}$ then constraint (7) is satisfied too and $\hat{x}$ maxinizes also (6) with constraint (7).

2. Suppose now that $x(0)$ violates the risk constraint (7). Then we consider the equivalent problem

$$\max_{x_i \geq 0} F(x)$$

$$R(x) \leq \alpha\bar{d}$$

Applying again the optimality conditions from [18] we obtain that its solution $\bar{x}$ takes the form $\bar{x} = x_{\mu^*}$ where x_μ, $\mu \geq 0$ solves the problem

$$\max_{x \geq 0} \left[F(x) - \mu R(x)\right] \quad (15)$$

and μ^* solves equation (10). Solution of (15) is characterized by conditions (13) where $F(x)$ is substituted by the objective in (15). Differentiating this objective as in (13) and utilizing again (14) we obtain that $x(\mu)$ from (9) is solution of (15) for fixed μ, $x(\mu) = x_\mu$. Then solution of equation (10) provides μ^* that yields $x(\mu^*)$, which maximizes (6) subject to (7). ■

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