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Exponential Companding and Active Constellation Extension Comparisons for PAPR Reduction

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***Abstract*—One of the major drawbacks of the orthogonal frequency division multiplexing (OFDM) type of communication is the high peak-to-average power ratio (PAPR) of the transmitted signal. Unless carefully tackled, this may cause either reduced energy efficiency or increased nonlinear distortion when the signal is amplified for transmission. For improving energy efficiency, this paper considers PAPR reduction in OFDM based systems. In particular, we study and compare some PAPR reduction approaches including three exponential companding and active constellation extension (ACE) methods. We discuss the trade-offs of the methods. In addition, for the ACE method defined in the ETSI standard, we propose some modifications that give about 1 dB more PAPR reduction without any additional power increase of the transmitted signal. Moreover, if some spectral regrowth due to the PAPR reduction is acceptable, exponential companding approaches can also be used.**

Keywords—ACE, active constellation extension, exponential companding, OFDM, peak-to-average power ratio

I. INTRODUCTION

The orthogonal frequency division multiplexing (OFDM) technique is widely used in many wireless communication standards [1]. It has benefits of high spectrum efficiency, robustness to the multipath fading, and low implementation complexity. One of the major drawbacks of the OFDM systems is the high peak-to-average power ratio (PAPR) of the transmitted signal. This may result in poor energy efficiency of the transmitter if the signal is simply backed-off to the linear region of the power amplifier (PA). Alternatively PAPR reduction and predistortion (PD) compensation of nonlinearities may be used to enable operation in the nonlinear region of the PA. Selecting a proper combination from these three approaches one needs to take into account power consumption in different parts of the transmitter [2].

There are a lot of PAPR reduction techniques for the OFDM systems proposed in the literature such as clipping and filtering, tone reservation, block coding, and partial transmit sequence techniques, see [3] and references therein.

The nonlinear companding method is one of the promising and extensively studied PAPR reduction method [3-7]. In the companding method, the dynamic range of the signal is compressed before transmission by nonlinear, usually exponential, operation. In the receiver side, it is possible to perform a de-companding operation to the received signal. The exponential companding method is attractive because it can be employed directly to any OFDM systems without any restrictions on its parameters [7]. In addition, by properly choosing the companding transform parameters, the average signal power does not change. However, this method is known to generate both in-band and out-of-band distortions that can be observed by increased bit error rate (BER) and adjacent channel power (ACP), respectively.

Another interesting PAPR reduction method is active constellation extension (ACE) [3, 8-10]. In the ACE method, the PAPR is reduced by appropriately extending outer boundaries of the signal constellations in frequency domain. When adjusting intelligently, the constellation modifications cancel the time domain peaks of the transmitted signal. This is achieved, in principle, with no degradation in the performance and only a little average power increase of the transmitted signal.

In this paper, we consider three different exponential companding methods. We compare their PAPR reduction performances and study the trade-offs between PAPR reduction, BER and ACP. Moreover, we consider the ACE method defined in one of the European Telecommunications Standards Institute (ETSI) Digital Video Broadcasting (DVB) standard [10]. Small modifications of the method are proposed.

The rest of the paper is organized as follows. The system model is introduced in Section II. Section III introduces the exponential companding as well as ACE methods. Simulation results are given in Section IV. Finally, conclusions are drawn in Section V.

II. SYSTEM MODEL

A simplified block diagram of the general OFDM based system with PAPR reduction is shown in Fig. 1. We use here a simplified long term evolution – advanced (LTE-A) downlink system. Either quadrature phase shift keying (QPSK) or 64 quadrature amplitude modulation (64QAM) symbols are used as input data. Then after OFDM modulation and upsampling to the 50 MHz sampling rate, PAPR of the signal is typically reduced. This is also valid when using exponential companding functions. However, the ACE PAPR reduction is performed inside the OFDM modulation block that is detailed more in the next section and Fig. 3. After possible PD and PA that are beyond scope of this paper, the signal is transmitted through an additive white Gaussian noise (AWGN) channel to a typical OFDM receiver.

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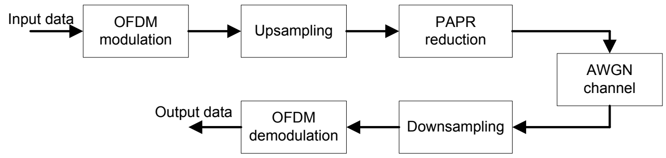

Fig. 1. Simplified block diagram of OFDM based system model.

III. PAPR REDUCTION METHODS

In this section, we first present three exponential companding methods, and then the ACE method and its extension are considered.

A. Exponential companding

Exponential companding method for reducing the PAPR of the OFDM based signal is proposed in [5]. It transforms Gaussian distributed OFDM signal into companded uniform distributed signal. Both small and large signal component are adjusted enabling to keep the average power level of the signal unchanged. The companding function [5] can be given by

$$h_1(x) = sgn(x)\sqrt[d]{\alpha\left[1 - e^{\left(\frac{x^2}{\sigma^2}\right)}\right]} \qquad (1)$$

where x is the input signal, σ^2 is the variance of the signal, and d is the degree of the first exponential companding scheme Parameter α is defined is defined as

$$\alpha = \left(\frac{E\left[|x|^2\right]}{E\left[\sqrt[d]{1-e^{\left(-\frac{|x|^2}{\sigma^2}\right)}}\right]}\right)^{d/2} \qquad (2)$$

where E[•] is the expectation value.

Another exponential companding method is proposed in [6]. Also this method maintains average power of the signal constant. The piecewise companding function [6] can be shown as follows

$$h_2(x) = \begin{cases} x & |x| \le \frac{\sigma}{\sqrt{6}} \\ sgn(x)\cdot\sqrt{6}\sigma\left(\frac{2}{3} - \frac{1}{2}e^{\left(\frac{1}{6}-\frac{|x|^2}{\sigma^2}\right)}\right) & |x| \ge \frac{\sigma}{\sqrt{6}} \end{cases} \qquad (3)$$

In [7], third similar but more flexible companding scheme is proposed. This method transforms the statistics of the OFDM signal into a specific distribution. By the adjustable parameters more design flexibility is achieved to satisfy different system requirements. The third piecewise exponential companding function [7] is shown in (4) at the bottom of the page. The parameters of (4) have following relationship

$$k = \frac{2(2\sigma c^2 - 2Ac + \sigma)}{\sigma(A - c\sigma^2)}e^{(-c^2)}. \qquad (5)$$

where A is the cutoff point of the target PDF (probability density function). Parameters k and c are used to specify the companding transform and to control the output power level.

There are also de-companding or expanding functions for the receiver side. These are not considered in this work as only a limited performance gain would be obtained [6-7] and simplification reduces complexity and power consumption.

B. Active constellation extension

As already mentioned, the idea behind the ACE method is to modify the signal constellations in frequency domain so that the PAPR of the time domain signal is reduced. This is obtained by appropriately extending outer boundaries of the signal constellations in frequency domain. Principles of the extensions for QPSK and 64QAM constellations are shown in Fig. 2. It can be clearly seen that the ACE method is more effective for small constellations because e.g. in the QPSK case all the subcarriers can be extended, while for 64QAM, only small part of the subcarriers can be affected. In principle, no degradation in the performance is assumed due to the constellation extensions but a little power increase.

In the ETSI standard [10], a simple non-iterative ACE method is considered. The illustration of the method is shown in Fig. 3. The method is controlled by a few parameters defined in the standard, namely “clipping threshold”, “gain”, and “extension limit”. The best performance is achieved by finding optimal set of the parameters with certain limits defined in [10].

In our studies we noticed that the limitation of the “extension limit” parameter restricts the PAPR reduction performance especially in 64QAM case, see Fig. 8. By allowing larger “extension limit”, more PAPR reduction without notable additional power increase is achieved. In addition, it is noticed that it could be worth to limit the power increase due to the use of ACE to a certain value. This could make link budget calculations easier for system designers.

$$h_3(x) = \begin{cases} x & |x| \le c\sigma \\ sgn(x)\cdot\frac{1}{k}\left(kc\sigma - \frac{2c}{\sigma}e^{-c^2} + \sqrt{\frac{4c^2}{\sigma^2}e^{-2c^2} + 2k\left(e^{-c^2} - e^{-\frac{|x|^2}{\sigma^2}}\right)}\right) & |x| \ge c\sigma \end{cases} \qquad (4)$$

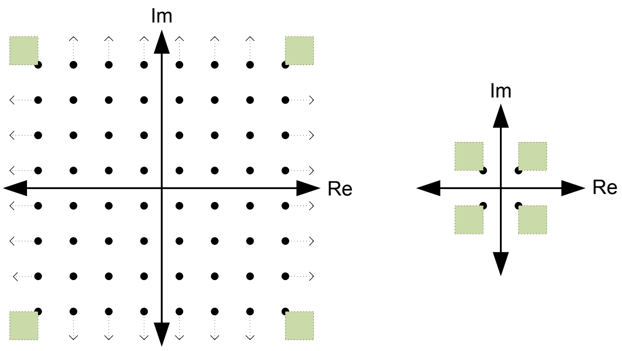

Fig. 2. Depiction of ACE method for 64QAM and QPSK constellations.

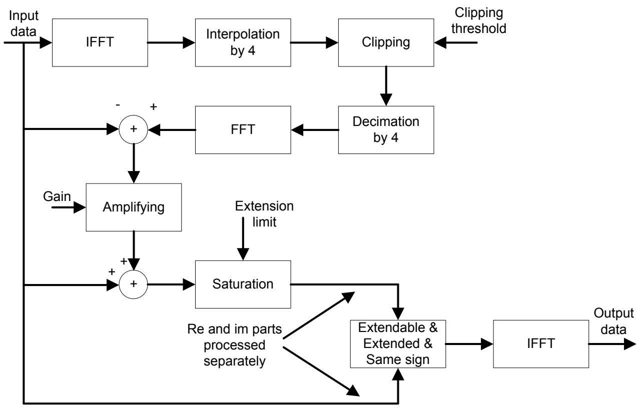

Fig. 3. Simplified implementation of ACE method defined in ETSI [10].

IV. RESULTS

In this section we demonstrate the performance of the studied PAPR reduction methods. We also highlight some essential points of the methods and compare them with each other.

As already explained in Section II, the simulations are based on the LTE-A system model. Constellations are set by resource block basis. In addition, there are no pilot subcarriers. Essential parameters are shown in Table I. For simplicity, the PD and PA are assumed as ideal and AWGN channel model is used in the simulations.

TABLE I. SIMULATION PARAMETERS

Parameters	**Values**
PDand PA	Assumed ideal
Signal bandwidth	5 and 20 MHz
FFT sizes	512 and 2048
Constellation sizes	QPSK and 64QAM
Number of used subcarries	300 and 1200
Cyclic prefix length	160 (1st symbol), 144 otherwise
Channel type	AWGN

A. Exponential companding

First, we illustrate achievable PAPR reductions of the exponential companding methods using 20 MHz signal. Fig. 4 shows complementary cumulative distribution functions (CCDFs) for the original as well as the PAPR reduced signals. Two first exponential companding methods reduce the PAPR a lot. These are inflexible methods as there is no possibility to control the amount of the PAPR reduction. The third method is more flexible via the parameters for setting different PAPR reduction levels. The case with about 3 dB PAPR reduction compared to the original signal at 10^{-2} probability is shown in Fig. 4. Note that no exhaustive search for optimal parameter sets of third companding method has been done, so it would be possible to find even better performance using the method.

As can be expected from the big differences of the PAPR reduction performances, also the spectral regrowth varies a lot between the companding methods, see Fig. 5. Again, when comparing to the ACE method, only the third companding method achieve reasonable spectral properties. It is well known that the ACE method does not increase the ACP, see Fig. 5.

Fig. 6 shows the BER versus SNR curves for the companded signals using 64QAM modulation over the AWGN channel. The first and second companding methods suffer huge performance degradation. This can also be expected from their very large PAPR reduction as seen in Fig. 4. Using QPSK modulation, the performance degradations are not so large. These results are not shown in the paper. Finally, we can conclude, that the third companding method achieves very good BER performance.

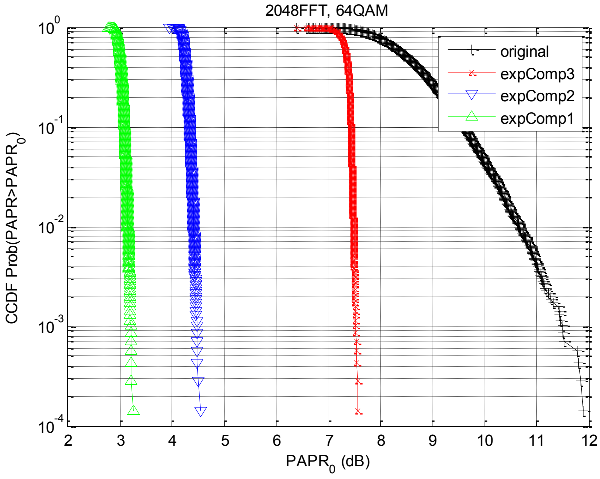

Fig. 4. CCDF of PAPR using exponential companding methods.

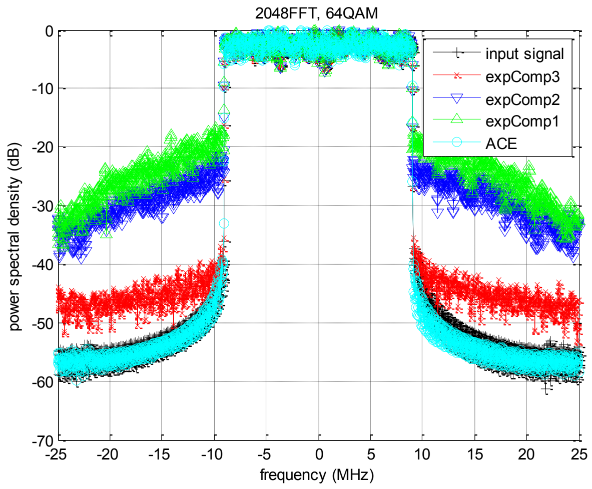

Fig. 5. Signal spectra before and after PAPR reduction.

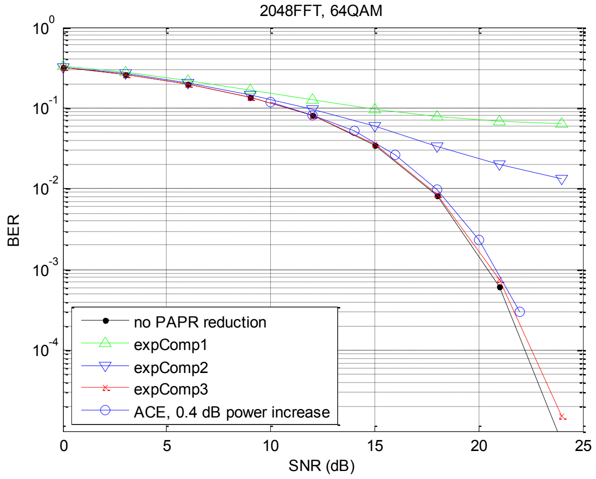

Fig. 6. BERs of the signals with and without PAPR reduction.

B. Active constellation extension

Considering the ACE method, we first study minimum achievable PAPR having certain limits for signal power increase. For 5 MHz signal with QPSK modulation minimum PAPR achieved is 6.2 dB meaning about 4 dB reduction of the original signal PAPR as shown in Fig. 7. Accepting 0.4 dB power increase, the PAPR reduction is still about 3 dB.

As can be seen in Fig. 8, for 20 MHz signal with 64QAM modulation absolute PAPR values are larger. Again, accepting 0.4 dB power increase, the PAPR is reduced about 3 dB. Note that, without notable effects to the signal power increase, about 1 dB improvement is achieved compared to the ACE method defined in ETSI standard [10]. In the standard, the “extension limit” parameter is allowed to be at maximum 1.4 dB whereas minimum PAPR is achieved having the “extension limit” about 2.2 dB.

Fig. 9 shows the CCDFs of the PAPR for the 20 MHz signal before and after the ACE PAPR reduction with 0.4 dB power increase. Considering the probability of 10^{-2}, the PAPR levels of 7 and 8 decibels are achieved for QPSK and 64QAM cases, respectively. Interestingly, if only 20 % of the data subcarriers are QPSK modulated and all others use 64QAM, additional PAPR reduction of 0.5 dB is achieved compared to the 64QAM only case.

It is assumed that the ACE method degrades the BER only with the amount close to the power increase, e.g. about 0.4 dB as in our simulations, see Fig. 6. These results assume that the SNR considers added transmit power by the larger noise variance.

When comparing the ACE and third exponential companding methods, we can conclude that about the same performances are achieved. The companding method increases the ACP only slightly, see Fig. 5. On the other hand, the ACE method increases also signal power in the defined limits. Considering shortly the complexities of the methods, the ACE method may be more complex, e.g. two additional FFT blocks are needed. On the other hand, part of the processing in ACE is done in frequency domain at a lower sampling rate. Complexity issues should be further studied in the future.

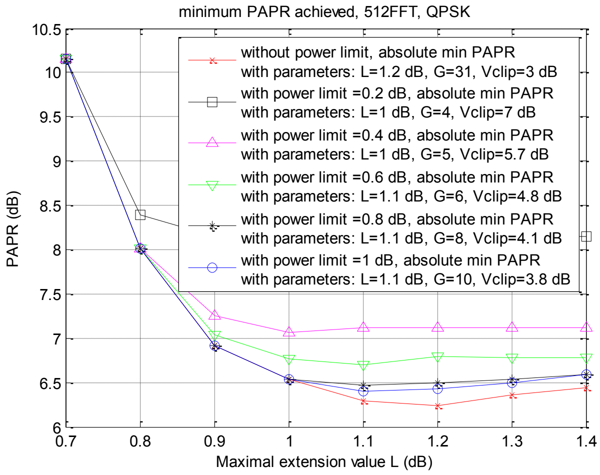

Fig. 7. Minimum PAPR achieved using ACE with different parameter settings.

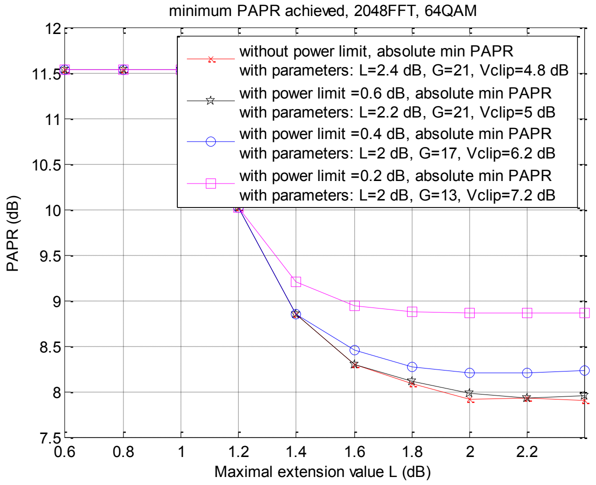

Fig. 8. Minimum PAPR achieved using ACE with different parameter settings.

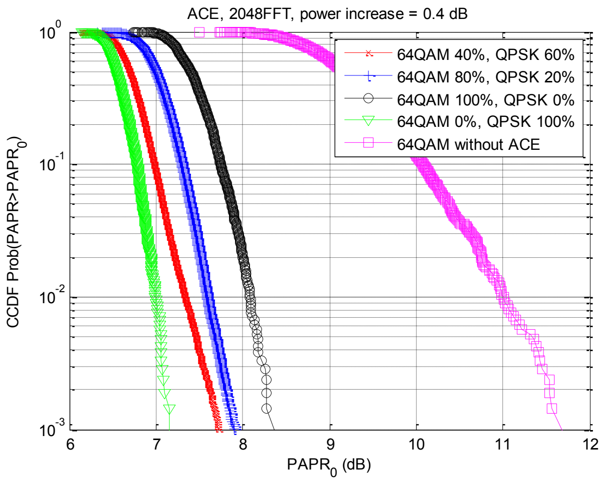

Fig. 9. PAPR CCDF using ACE with different combination of 64QAM and QPSK modulations.

V. CONCLUSION

In this paper, we studied the PAPR reduction in the OFDM based systems. In particular, we studied and compared some PAPR reduction approaches including the three exponential companding and the ACE methods. We discussed the trade-offs of the methods. In addition, for the ACE method defined in ETSI standard, we proposed some modifications that give about 1 dB more PAPR reduction without notable additional power increase of the transmitted signal. Moreover, we proposed that the power increase due to the use of ACE method could be limited to a certain value (e.g. 0.4 dB). The ACE and third exponential companding methods achieve comparable performances in terms of the BER, PAPR reduction, and ACP.

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